HL Paper 2

A continuous random variable X has a probability density function given by the function $f(x)$, where

$$f(x) = \begin{cases} k(x+2)^2, & -2 \leqslant x < 0 \\ k, & 0 \leqslant x \leqslant \frac{4}{3} \\ 0, & \text{otherwise.} \end{cases}$$

a. Find the value of k . [2]

b. Hence find [5]

(i) the mean of X ;

(ii) the median of X .

Markscheme

a. $k\int_{-2}^{0}(x+2)^2\mathrm{d}x + \int_{0}^{\frac{4}{3}}k\mathrm{d}x = 1$ ***M1***

$\frac{8k}{3} + \frac{4k}{3} = 1$

$k = \frac{1}{4}$ ***A1***

Note: Only ft on positive values of k.

[2 marks]

b. (i) $\mathrm{E}(X) = \frac{1}{4}\int_{-2}^{0}x(x+2)^2\mathrm{d}x + \frac{1}{4}\int_{0}^{\frac{4}{3}}x\mathrm{d}x$ ***M1***

$= \frac{1}{4} \times \frac{-4}{3} + \frac{2}{9}$

$= -\frac{1}{9}$ (-0.111) ***A1***

(ii) median given by a such that $\mathrm{P}(X < a) = 0.5$

$\frac{1}{4}\int_{-2}^{a}(x+2)^2\mathrm{d}x = 0.5$ **M1**

$\left[\frac{(x+2)^3}{3}\right]_{-2}^{a} = 2$ ***(A1)***

$(a+2)^3 - 0 = 6$

$a = \sqrt[3]{6} - 2$ $(= -0.183)$ ***A1***

[5 marks]

Examiners report

a. Many candidates recognised that integration was the appropriate technique to solve this question but the fact that the function was piecewise proved problematic for many. Good use of technology by some candidates was seen but few sketches of the function were made. A sketch would have been helpful to many candidates when attempting to solve (b (ii).

b. Many candidates recognised that integration was the appropriate technique to solve this question but the fact that the function was piecewise proved problematic for many. Good use of technology by some candidates was seen but few sketches of the function were made. A sketch would have been helpful to many candidates when attempting to solve (b (ii).

The events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.48$ and $\mathrm{P}(A \cup B) = 0.818$.

a. Find $\mathrm{P}(A \cap B)$. [2]

b. Hence show that the events A and B are independent. [2]

Markscheme

a. **Note:** In Section A, where appropriate, accept answers that correctly round to 2 sf except in Q2, Q5(a) (ii), Q5(b) and Q8(a).

$0.818 = 0.65 + 0.48 - \mathrm{P}(A \cap B)$ ***(M1)***

$\mathrm{P}(A \cap B) = 0.312$ ***A1***

[2 marks]

b. $\mathrm{P}(A)P(B) = 0.312 \quad (= 0.48 \times 0.65)$ ***A1***

since $\mathrm{P}(A)P(B) = \mathrm{P}(A \cap B)$ then A and B are independent ***R1***

Note: Only award the ***R1*** if numerical values are seen. Award ***A1R1*** for a correct conditional probability approach.

[2 marks]

Total [4 marks]

Examiners report

a. [N/A]
b. [N/A]

The probability that the 08:00 train will be delayed on a work day (Monday to Friday) is $\frac{1}{10}$. Assuming that delays occur independently,

a. find the probability that the 08:00 train is delayed exactly twice during any period of five work days; [2]

b. find the minimum number of work days for which the probability of the 08:00 train being delayed at least once exceeds 90 %. [3]

Markscheme

a. $X \sim \mathrm{B}(5, 0.1)$ ***(M1)***

$\mathrm{P}(X = 2) = 0.0729$ ***A1***

[2 marks]

b. $\mathrm{P}(X \geqslant 1) = 1 - \mathrm{P}(X = 0)$ ***(M1)***

$0.9 < 1 - \left(\frac{9}{10}\right)^n$ ***(M1)***

$n > \frac{\ln 0.1}{\ln 0.9}$

$n = 22$ days ***A1***

[3 marks]

Examiners report

a. This question was generally answered successfully. Many candidates used the tabular feature of their GDC for (b) thereby avoiding potential errors in the algebraic manipulation of logs and inequalities.

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The weight loss, in kilograms, of people using the slimming regime *SLIM3M* for a period of three months is modelled by a random variable X.

Experimental data showed that 67 % of the individuals using *SLIM3M* lost up to five kilograms and 12.4 % lost at least seven kilograms.

Assuming that X follows a normal distribution, find the expected weight loss of a person who follows the *SLIM3M* regime for three months.

Markscheme

$X \sim \mathrm{N}(\mu,\ \sigma^2)$

$\mathrm{P}(X \leqslant 5) = 0.670 \Leftrightarrow \frac{5-\mu}{\sigma} = 0.4399\ldots$ ***M1A1***

$\mathrm{P}(X > 7) = 0.124 \Leftrightarrow \frac{7-\mu}{\sigma} = 1.155\ldots$ ***A1***

solve simultaneously

$\mu + 0.4399\sigma = 5$ and $\mu + 1.1552\sigma = 7$ ***M1***

$\mu = 3.77$ (3 sf) ***A1*** ***N3***

the expected weight loss is 3.77 kg

Note: Award ***A0*** for $\mu = 3.78$ (answer obtained due to early rounding).

[5 marks]

Examiners report

Although many candidates were successful in answering this question, a surprising number of candidates did not even attempt it. The main difficulty was in finding the correct z scores. A fairly common error was to misinterpret one of the conditions and obtain one of the equations as $\frac{7-\mu}{\sigma} = -1.155\ldots$. In some cases candidates failed to keep the accuracy throughout the question and obtained inaccurate answers.

The random variable X has the distribution $\mathrm{B}(30,\ p)$. Given that $\mathrm{E}(X) = 10$, find

a. the value of p ; [1]

b. $\mathrm{P}(X = 10)$; [2]

c. $\mathrm{P}(X \geqslant 15)$. [2]

Markscheme

a. $\mathrm{E}(X) = np$

$\Rightarrow 10 = 30p$

$\Rightarrow p = \frac{1}{3}$ ***A1***

[1 mark]

b. $P(X = 10) = \binom{30}{10}\left(\frac{1}{3}\right)^{10}\left(\frac{2}{3}\right)^{20} = 0.153$ ***(M1)A1***

[2 marks]

c. $\mathrm{P}(X \geqslant 15) = 1 - \mathrm{P}(X \leqslant 14)$ ***(M1)***

$= 1 - 0.9565... = 0.0435$ ***A1***

[2 marks]

Examiners report

a. Again this proved to be an accessible question for students with many students gaining full marks. Most candidates used the calculator to find the answers to parts (b) and (c) which is what was intended, but candidates should be aware that there are often marks for recognising what needs to be found, even if the candidate does not obtain the final correct answer. It is suggested that in this style of question, candidates should indicate what they are trying to find as well as giving the final answer.

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The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases} 0,\ x<0 \\ \frac{\sin x}{4},\ 0\le x\le\pi \\ a(x-\pi),\ \pi<x\le 2\pi \\ 0,\ 2\pi<x \end{cases}.$$

a. Sketch the graph $y=f(x)$. [2]

b. Find $\mathrm{P}(X\le\pi)$. [2]

c. Show that $a=\frac{1}{\pi^2}$. [3]

d. Write down the median of X. [1]

e. Calculate the mean of X. [3]

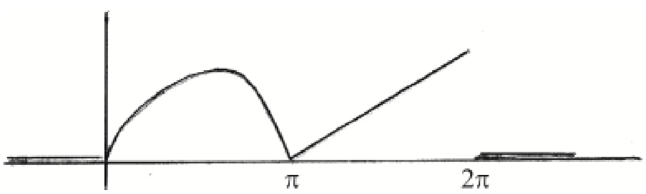
f. Calculate the variance of X. [3]

g. Find $\mathrm{P}\left(\frac{\pi}{2}\le X\le\frac{3\pi}{2}\right)$. [2]

h. Given that $\frac{\pi}{2}\le X\le\frac{3\pi}{2}$ find the probability that $\pi\le X\le 2\pi$. [4]

Markscheme

a.



Award ***A1*** for sine curve from 0 to π, award ***A1*** for straight line from π to 2π ***A1A1***

[2 marks]

b. $\int_0^\pi \frac{\sin x}{4}\mathrm{d}x=\frac{1}{2}$ ***(M1)A1***

[2 marks]

c. **METHOD 1**

require $\frac{1}{2}+\int_\pi^{2\pi} a(x-\pi)\mathrm{d}x=1$ ***(M1)***

$\Rightarrow \frac{1}{2}+a\left[\frac{(x-\pi)^2}{2}\right]_\pi^{2\pi}=1\quad \left(\text{or } \frac{1}{2}+a\left[\frac{x^2}{2}-\pi x\right]_\pi^{2\pi}=1\right)$ ***A1***

$\Rightarrow a\frac{\pi^2}{2}=\frac{1}{2}$ ***A1***

$\Rightarrow a=\frac{1}{\pi^2}$ ***AG***

Note: Must obtain the exact value. Do not accept answers obtained with calculator.

METHOD 2

$0.5 + \text{ area of triangle } = 1$ ***R1***

area of triangle $= \frac{1}{2}\pi \times a\pi = 0.5$ ***M1A1***

Note: Award ***M1*** for correct use of area formula $= 0.5$, ***A1*** for $a\pi$.

$a = \frac{1}{\pi^2}$ ***AG***

[3 marks]

d. median is π ***A1***

[1 mark]

e. $\mu = \int_0^{\pi} x \cdot \frac{\sin x}{4}\mathrm{d}x + \int_{\pi}^{2\pi} x \cdot \frac{x-\pi}{\pi^2}\mathrm{d}x$ ***(M1)(A1)***

$= 3.40339\ldots = 3.40 \quad \left(\text{or } \frac{\pi}{4} + \frac{5\pi}{6} = \frac{13}{12}\pi\right)$ ***A1***

[3 marks]

f. For $\mu = 3.40339\ldots$

EITHER

$\sigma^2 = \int_0^{\pi} x^2 \cdot \frac{\sin x}{4}\mathrm{d}x + \int_{\pi}^{2\pi} x^2 \cdot \frac{x-\pi}{\pi^2}\mathrm{d}x - \mu^2$ ***(M1)(A1)***

OR

$\sigma^2 = \int_0^{\pi} (x-\mu)^2 \cdot \frac{\sin x}{4}\mathrm{d}x + \int_{\pi}^{2\pi} (x-\mu)^2 \cdot \frac{x-\pi}{\pi^2}\mathrm{d}x$ ***(M1)(A1)***

THEN

$= 3.866277\ldots = 3.87$ ***A1***

[3 marks]

g. $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{4}\mathrm{d}x + \int_{\pi}^{\frac{3\pi}{2}} \frac{x-\pi}{\pi^2}\mathrm{d}x = 0.375 \quad \left(\text{or } \frac{1}{4} + \frac{1}{8} = \frac{3}{8}\right)$ ***(M1)A1***

[2 marks]

h. $\mathrm{P}\left(\pi \le X \le 2\pi \,\middle|\, \frac{\pi}{2} \le X \le \frac{3\pi}{2}\right) = \frac{\mathrm{P}\left(\pi \le X \le \frac{3\pi}{2}\right)}{\mathrm{P}\left(\frac{\pi}{2} \le X \le \frac{3\pi}{2}\right)}$ ***(M1)(A1)***

$= \frac{\int_{\pi}^{\frac{3\pi}{2}} \frac{(x-\pi)}{\pi^2}\mathrm{d}x}{0.375} = \frac{0.125}{0.375} \quad \left(\text{or } = \frac{\frac{1}{8}}{\frac{3}{8}} \text{ from diagram areas}\right)$ ***(M1)***

$= \frac{1}{3} \quad (0.333)$ ***A1***

[4 marks]

Total [20 marks]

Examiners report

a. Most candidates sketched the graph correctly. In a few cases candidates did not seem familiar with the shape of the graphs and ignored the fact that the graph represented a pdf. The correct sketch assisted greatly in the rest of the question.

b. Most candidates answered this question correctly.

c. A few good proofs were seen but also many poor answers where the candidates assumed what you were trying to prove and verified numerically the result.

d. Most candidates stated the value correctly but many others showed no understanding of the concept.

e. Many candidates scored full marks in this question; many others could not apply the formula due to difficulties in dealing with the piecewise function. For example, a number of candidates divided the final answer by two.

f. Many misconceptions were identified: use of incorrect formula (e.g. formula for discrete distributions), use of both expressions as integrand and division of the result by 2 at the end.

g. This part was fairly well done with many candidates achieving full marks.

h. Many candidates had difficulties with this part showing that the concept of conditional probability was poorly understood. The best candidates did it correctly from the sketch.

A ski resort finds that the mean number of accidents on any given weekday (Monday to Friday) is 2.2 . The number of accidents can be modelled by a Poisson distribution.

a(i)(ii). Find the probability that in a certain week (Monday to Friday only) [6]

(i) there are fewer than 12 accidents;
(ii) there are more than 8 accidents, given that there are fewer than 12 accidents.

b(i)(ii). Due to the increased usage, it is found that the probability of more than 3 accidents in a day at the weekend (Saturday and Sunday) is 0.24. [10]

Assuming a Poisson model,
(i) calculate the mean number of accidents per day at the weekend (Saturday and Sunday);
(ii) calculate the probability that, in the four weekends in February, there will be more than 5 accidents during at least two of the weekends.

c. It is found that 20 % of skiers having accidents are at least 25 years of age and 40 % are under 18 years of age. [6]

Assuming that the ages of skiers having accidents are normally distributed, find the mean age of skiers having accidents.

Markscheme

a(i)(ii). (i) $X \sim \mathrm{Po}(11)$ ***(M1)***

$\mathrm{P}(X \leqslant 11) = 0.579$ ***(M1)A1***

(ii) $\mathrm{P}(X > 8 \,|\, x < 12) =$ ***(M1)***

$= \frac{\mathrm{P}(8<X<12)}{\mathrm{P}(X<12)} \left(\text{or } \frac{\mathrm{P}(X\leqslant 11)-\mathrm{P}(X\leqslant 8)}{\mathrm{P}(X\leqslant 11)} \text{ or } \frac{0.3472...}{0.5792...}\right)$ ***A1***

$= 0.600$ ***A1 N2***

[6 marks]

b(i)(ii). (i) $Y \sim \mathrm{Po}(m)$

$\mathrm{P}(Y > 3) = 0.24$ ***(M1)***

$\mathrm{P}(Y \leqslant 3) = 0.76$ ***(M1)***

$\mathrm{e}^{-m}\left(1 + m + \frac{1}{2}m^2 + \frac{1}{6}m^3\right) = 0.76$ ***(A1)***

Note: At most two of the above lines can be implied.

Attempt to solve equation with GDC ***(M1)***

$m = 2.49$ ***A1***

(ii) $A \sim \mathrm{Po}(4.98)$

$\mathrm{P}(A > 5) = 1 - \mathrm{P}(A \leqslant 5) = 0.380...$ ***M1A1***

$W \sim \mathrm{B}(4,\ 0.380...)$ ***(M1)***

$\mathrm{P}(W \geqslant 2) = 1 - \mathrm{P}(W \leqslant 1) = 0.490$ ***M1A1***

[10 marks]

c. $\mathrm{P}(A < 25) = 0.8,\ \mathrm{P}(A < 18) = 0.4$

$\frac{25-\mu}{\sigma} = 0.8416...$ ***(M1)(A1)***

$\frac{18-\mu}{\sigma} = -0.2533...\ (\text{or} - 0.2534 \text{ from tables})$ ***(M1)(A1)***

solving these equations ***(M1)***

$\mu = 19.6$ ***A1***

Note: Accept just 19.6, 19 or 20; award A0 to any other final answer.

[6 marks]

Examiners report

a(i)(ii). Generally, candidates had difficulties with this question, mainly in applying conditional probability and interpreting the expressions ‘more than’, ‘at least’ and ‘under’ to obtain correct expressions. Although many candidates identified the binomial distribution in part (b) (ii), very few succeeded in answering this question due to incorrect interpretation of the question or due to accuracy errors.

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The probability density function of a continuous random variable X is given by

$f(x) = \frac{1}{1+x^4}$, 0 ″ x ″ a .

a. Find the value of a . [3]

b. Find the mean of X. [2]

Markscheme

a. $\int_0^a \frac{1}{1+x^4} \mathrm{d}x = 1$ ***M2***

$a = 1.40$ ***A1***

[3 marks]

b. $\mathrm{E}(X) = \int_0^a \frac{x}{1+x^4} \mathrm{d}x$ ***M1***

$\left(= \frac{1}{2} \arctan(a^2)\right)$

$= 0.548$ ***A1***

[2 marks]

Examiners report

a. Many candidates picked up some marks for this question, but only a few gained full marks. In part (a) many candidates did not appreciate the need for the calculator to find a value of a. Candidates had more success with part (b) with a number of candidates picking up follow through marks.

b. Many candidates picked up some marks for this question, but only a few gained full marks. In part (a) many candidates did not appreciate the need for the calculator to find a value of a. Candidates had more success with part (b) with a number of candidates picking up follow through marks.

The length, X metres, of a species of fish has the probability density function

$$f(x) = \begin{cases} ax^2, & \text{for } 0 \leqslant x \leqslant 0.5 \\ 0.5a(1-x), & \text{for } 0.5 \leqslant x \leqslant 1 \\ 0, & \text{otherwise .} \end{cases}$$

a. Show that $a = 9.6$. [3]

b. Sketch the graph of the distribution. [2]

c. Find $\mathrm{P}(X < 0.6)$. [2]

Markscheme

a. $\int_0^{0.5} ax^2 \mathrm{d}x + \int_{0.5}^1 0.5a(1-x) \mathrm{d}x = 1$ ***M1A1***

$\frac{5a}{48}$ (or equivalent) or $a \times 0.104\ldots = 1$ ***A1***

Note: Award ***M1*** for considering two definite integrals.

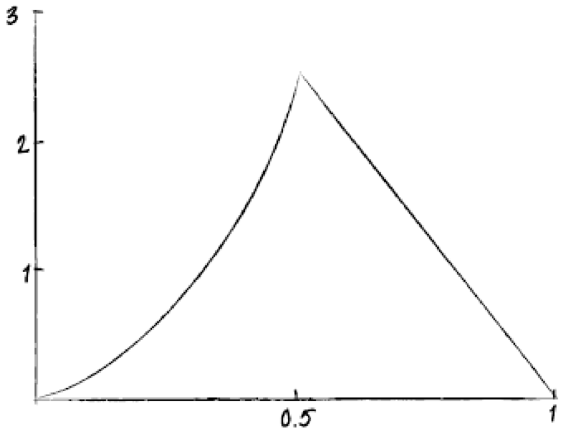
Award ***A1*** for equating to 1.

Award ***A1*** for a correct equation.

The ***A1A1*** can be awarded in any order.

$a = 9.6$ ***AG***

[3 marks]

b. 

correct shape for $0 \leqslant x \leqslant 0.5$ and $f(0.5) \approx 2.4$ ***A1***

correct shape for $0.5 \leqslant x \leqslant 1$ and $f(1) = 0$ ***A1***

[2 marks]

c. attempting to find $\mathrm{P}(X < 0.6)$ ***(M1)***

direct GDC use or *eg* $\mathrm{P}(0 \leqslant X \leqslant 0.5) + \mathrm{P}(0.5 \leqslant X \leqslant 0.6)$ or $1 - \mathrm{P}(0.6 \leqslant X \leqslant 1)$

$\mathrm{P}(X < 0.6) = 0.616 \left(= \frac{77}{125}\right)$ ***A1***

[2 marks]

Examiners report

a. Part (a) was generally well done. Common errors usually involved not recognizing that the sum of the two integrals was equal to one, premature rounding or not showing full working to conclusively show that $a = 9.6$.

b. Part (b) was not well done with many graphs poorly labelled and offering no reference to domain and range.

c. Part (c) was reasonably well done. The most common error involved calculating an incorrect probability from an incorrect definite integral.

The number of accidents that occur at a large factory can be modelled by a Poisson distribution with a mean of 0.5 accidents per month.

a. Find the probability that no accidents occur in a given month. [1]

b. Find the probability that no accidents occur in a given 6 month period. [2]

c. Find the length of time, in complete months, for which the probability that at least 1 accident occurs is greater than 0.99. [6]

d. To encourage safety the factory pays a bonus of \$1000 into a fund for workers if no accidents occur in any given month, a bonus of \$500 if 1 or 2 accidents occur and no bonus if more than 2 accidents occur in the month. [9]

(i) Calculate the expected amount that the company will pay in bonuses each month.

(ii) Find the probability that in a given 3 month period the company pays a total of exactly \$2000 in bonuses.

Markscheme

a. $\mathrm{P}(x = 0) = 0.607$ ***A1***

[1 mark]

b. **EITHER**

Using $X \sim \mathrm{Po}(3)$ ***(M1)***

OR

Using $(0.6065\ldots)^6$ ***(M1)***

THEN

$\mathrm{P}(X = 0) = 0.0498$ ***A1***

[2 marks]

c. $X \sim \mathrm{Po}(0.5t)$ ***(M1)***

$\mathrm{P}(x \geqslant 1) = 1 - \mathrm{P}(x = 0)$ ***(M1)***

$\mathrm{P}(x = 0) < 0.01$ ***A1***

$\mathrm{e}^{-0.5t} < 0.01$ ***A1***

$-0.5t < \ln(0.01)$ ***(M1)***

$t > 9.21$ months

therefore 10 months ***A1N4***

Note: Full marks can be awarded for answers obtained directly from GDC if a systematic method is used and clearly shown.

[6 marks]

d. (i) P(1 or 2 accidents) = 0.37908… ***A1***

$\mathrm{E}(B) = 1000 \times 0.60653... + 500 \times 0.37908\ldots$ ***M1A1***

$= \$796$ (accept \$797 or \$796.07) ***A1***

(ii) P(2000) = P(1000, 1000, 0) + P(1000, 0, 1000) + P(0, 1000, 1000) +

P(1000, 500, 500) + P(500, 1000, 500) + P(500, 500, 1000) ***(M1)(A1)***

Note: Award ***M1*** for noting that 2000 can be written both as $2 \times 1000 + 1 \times 0$ and $2 \times 500 + 1 \times 1000$.

$= 3(0.6065...)^2(0.01437...) + 3(0.3790...)^2(0.6065\ldots)$ ***M1A1***

$= 0.277$ (accept 0.278) ***A1***

[9 marks]

Examiners report

a. Most candidates successfully answered (a) and (b). Although many found the correct answer to (c), communication of their reasoning was weak. This was also true for (d)(i). Answers to (d)(ii) were mostly scrappy and rarely worthy of credit.

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The number of visitors that arrive at a museum every minute can be modelled by a Poisson distribution with mean 2.2.

a. If the museum is open 6 hours daily, find the expected number of visitors in 1 day. [2]

b. Find the probability that the number of visitors arriving during an hour exceeds 100. [3]

c. Find the probability that the number of visitors in each of the 6 hours the museum is open exceeds 100. [2]

d. The ages of the visitors to the museum can be modelled by a normal distribution with mean μ and variance σ^2 . The records show that 29 % of the visitors are under 35 years of age and 23 % are at least 55 years of age. [6]
Find the values of μ and σ .

e. The ages of the visitors to the museum can be modelled by a normal distribution with mean μ and variance σ^2 . The records show that 29 % of the visitors are under 35 years of age and 23 % are at least 55 years of age. [5]
One day, 100 visitors under 35 years of age come to the museum. Estimate the number of visitors under 50 years of age that were at the museum on that day.

Markscheme

a. $2.2 \times 6 \times 60 = 792$ ***(M1)A1***

[2 marks]

b. $V \sim \mathrm{Po}(2.2 \times 60)$ ***(M1)***

$\mathrm{P}(V > 100) = 0.998$ ***(M1)A1***

[3 marks]

c. $(0.997801...)^6 = 0.987$ ***(M1)A1***

[2 marks]

d.

$A \sim \mathrm{N}(\mu,\, \sigma^2)$
$\mathrm{P}(A < 35) = 0.29$ and $\mathrm{P}(A > 55) = 0.23 \Rightarrow \mathrm{P}(A < 55) = 0.77$
$\mathrm{P}\left(Z < \frac{35-\mu}{\sigma}\right) = 0.29$ and $\mathrm{P}\left(Z < \frac{55-\mu}{\sigma}\right) = 0.77$ ***(M1)***
use of inverse normal ***(M1)***
$\frac{35-\mu}{\sigma} = -0.55338...$ and $\frac{55-\mu}{\sigma} = 0.738846...$ ***(A1)***
solving simultaneously ***(M1)***

$\mu = 43.564...$ and $\sigma = 15.477...$ ***A1A1***
$\mu = 43.6$ and $\sigma = 15.5$ (3sf)
[6 marks]

e. $0.29n = 100 \Rightarrow n = 344.82...$ ***(M1)(A1)***

$\mathrm{P}(A < 50) = 0.66121...$ ***(A1)***
expected number of visitors under 50 = 228 ***(M1)A1***
[5 marks]

Examiners report

a. This question was generally well done by most candidates. It was evident that candidates had been well prepared in Poisson and normal distribution. In parts (a)-(d) candidates were usually successful and appropriate methods were shown although many candidates used labored algebraic approaches to solving simultaneous equations and wasted time answering part (d). Part (e) was very well answered by a smaller number of candidates but it was obviously more demanding in its level of abstraction.

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A continuous random variable X has probability density function f given by

$f(x) = \begin{cases} \frac{x^2}{a} + b, & 0 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$ where a and b are positive constants.

It is given that $\mathrm{P}(X \geqslant 2) = 0.75$.

Eight independent observations of X are now taken and the random variable Y is the number of observations such that $X \geqslant 2$.

a. Show that $a=32$ and $b=\frac{1}{12}$. [5]

b. Find $\mathrm{E}(X)$. [2]

c. Find $\mathrm{Var}(X)$. [2]

d. Find the median of X. [3]

e. Find $\mathrm{E}(Y)$. [2]

f. Find $\mathrm{P}(Y \geqslant 3)$. [1]

Markscheme

a. $\int_0^4 \left(\frac{x^2}{a}+b\right) \mathrm{d}x = 1 \Rightarrow \left[\frac{x^3}{3a}+bx\right]_0^4 = 1 \Rightarrow \frac{64}{3a}+4b=1$ ***M1A1***

$\int_2^4 \left(\frac{x^2}{a}+b\right) \mathrm{d}x = 0.75 \Rightarrow \frac{56}{3a}+2b=0.75$ ***M1A1***

Note: $\int_0^2 \left(\frac{x^2}{a}+b\right) dx = 0.25 \Rightarrow \frac{8}{3a}+2b=0.25$ could be seen/used in place of either of the above equations.

evidence of an attempt to solve simultaneously (or check given *a*,*b* values are consistent) ***M1***

$a=32,\ b=\frac{1}{12}$ ***AG***

[5 marks]

b. $\mathrm{E}\left(X\right)=\int_0^4 x\left(\frac{x^2}{32}+\frac{1}{12}\right)\mathrm{d}x$ ***(M1)***

$\mathrm{E}(X)=\frac{8}{3}\ (=2.67)$ ***A1***

[2 marks]

c. $\mathrm{E}\left(X^2\right)=\int_0^4 x^2\left(\frac{x^2}{32}+\frac{1}{12}\right)\mathrm{d}x$ ***(M1)***

$\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2=\frac{16}{15}\ (=1.07)$ ***A1***

[2 marks]

d. $\int_0^m \left(\frac{x^2}{32}+\frac{1}{12}\right)\mathrm{d}x=0.5$ ***(M1)***

$\frac{m^3}{96}+\frac{m}{12}=0.5\ (\Rightarrow m^3+8m-48=0)$ ***(A1)***

$m=2.91$ ***A1***

[3 marks]

e. $Y \sim B(8,\ 0.75)$ ***(M1)***

$\mathrm{E}(Y)=8\times 0.75=6$ ***A1***

[2 marks]

f. $\mathrm{P}(Y \geqslant 3)=0.996$ ***A1***

[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

John likes to go sailing every day in July. To help him make a decision on whether it is safe to go sailing he classifies each day in July as windy or calm. Given that a day in July is calm, the probability that the next day is calm is 0.9. Given that a day in July is windy, the probability that the next day is calm is 0.3. The weather forecast for the 1st July predicts that the probability that it will be calm is 0.8.

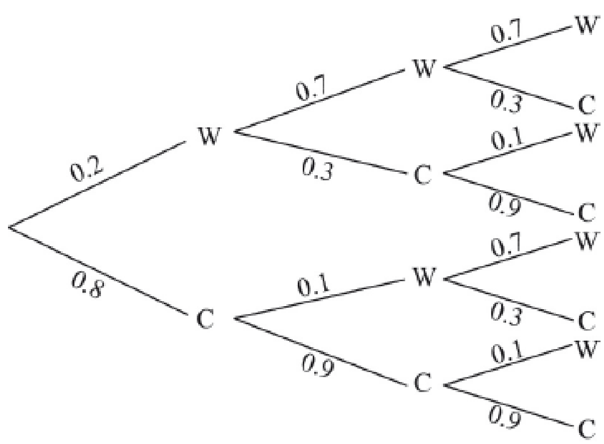
a. Draw a tree diagram to represent this information for the first three days of July. [3]

b. Find the probability that the 3rd July is calm. [2]

c. Find the probability that the 1st July was calm given that the 3rd July is windy. [4]

Markscheme

a.



M1A2

Note: Award ***M1*** for 3 stage tree-diagram, ***A2*** for 0.8, 0.9, 0.3 probabilities correctly placed.

[3 marks]

b. $0.2 \times 0.7 \times 0.3 + 0.2 \times 0.3 \times 0.9 + 0.8 \times 0.1 \times 0.3 + 0.8 \times 0.9 \times 0.9 = 0.768$ ***(M1)A1***

[2 marks]

c. $\mathrm{P(1st\ July\ is\ calm \mid 3rd\ July\ is\ windy)} = \frac{\mathrm{P(1st\ July\ is\ calm\ and\ 3rd\ July\ is\ windy)}}{\mathrm{P(3rd\ July\ is\ windy)}}$ ***(M1)***

$= \frac{0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}{1 - 0.768}$

OR $\frac{0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}{0.2 \times 0.7 \times 0.7 + 0.2 \times 0.3 \times 0.1 + 0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}$

OR $\frac{0.128}{0.232}$ ***(A1)(A1)***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

$= 0.552$ ***A1***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the data set $\{k-2,\ k,\ k+1,\ k+4\}$, where $k \in \mathbb{R}$.

(a) Find the mean of this data set in terms of *k*.

Each number in the above data set is now **decreased** by 3.

(b) Find the mean of this **new** data set in terms of *k*.

Markscheme

(a) Use of $\bar{x} = \frac{\sum_{i=1}^{4} x_i}{n}$ ***(M1)***

$\bar{x} = \frac{(k-2)+k+(k+1)+(k+4)}{4}$ ***(A1)***

$\bar{x} = \frac{4k+3}{4} \quad \left(= k + \frac{3}{4}\right)$ ***A1 N3***

(b) Either attempting to find the new mean or subtracting 3 from **their** $\bar{x}$ ***(M1)***

$\bar{x} = \frac{4k+3}{4} - 3 \quad \left(= \frac{4k-9}{4},\ k - \frac{9}{4}\right)$ ***A1 N2***

[5 marks]

Examiners report

This was an easy question that was well done by most candidates. Careless arithmetic errors caused some candidates not to earn full marks. Only a few candidates realised that part (b) could be answered correctly by directly subtracting 3 from their answer to part (a). Most successful responses were obtained by redoing the calculation from part (a).

Six balls numbered 1, 2, 2, 3, 3, 3 are placed in a bag. Balls are taken one at a time from the bag at random and the number noted. Throughout the question a ball is always replaced before the next ball is taken.

Three balls are taken from the bag. Find the probability that

a. A single ball is taken from the bag. Let X denote the value shown on the ball. [2]

Find $\mathrm{E}(X)$.

b.i. the total of the three numbers is 5; [3]

b.ii. the median of the three numbers is 1. [3]

c. Ten balls are taken from the bag. Find the probability that less than four of the balls are numbered 2. [3]

d. Find the least number of balls that must be taken from the bag for the probability of taking out at least one ball numbered 2 to be greater than 0.95. [3]

e. Another bag also contains balls numbered 1 , 2 or 3. [8]

Eight balls are to be taken from this bag at random. It is calculated that the expected number of balls numbered 1 is 4.8 , and the variance of the number of balls numbered 2 is 1.5.

Find the least possible number of balls numbered 3 in this bag.

Markscheme

a. $\mathrm{E}(X) = 1 \times \frac{1}{6} + 2 \times \frac{2}{6} + 3 \times \frac{3}{6} = \frac{14}{6} \left(= \frac{7}{3} = 2.33\right)$ ***(M1)A1***

[2 marks]

b.i. $3 \times \mathrm{P}(113) + 3 \times \mathrm{P}(122)$ ***(M1)***

$3 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{2} + 3 \times \frac{1}{6} \times \frac{1}{3} \times \frac{1}{3} = \frac{7}{72} \ (= 0.0972)$ ***A1***

Note: Award ***M1*** for attempt to find at least four of the cases.

[3 marks]

b.ii. recognising 111 as a possibility $\left(\text{implied by } \frac{1}{216}\right)$ ***(M1)***

recognising 112 and 113 as possibilities $\left(\text{implied by } \frac{2}{216} \text{ and } \frac{3}{216}\right)$ ***(M1)***

seeing the three arrangements of 112 and 113 ***(M1)***

$\mathrm{P}(111) + 3 \times \mathrm{P}(112) + 3 \times \mathrm{P}(113)$

$= \frac{1}{216} + \frac{6}{216} + \frac{9}{216} = \frac{16}{216} \left(= \frac{2}{27} = 0.0741\right)$ ***A1***

[3 marks]

c. let the number of twos be $X,\ X \sim B\left(10,\ \frac{1}{3}\right)$ ***(M1)***

$\mathrm{P}(X < 4) = \mathrm{P}(X \leqslant 3) = 0.559$ ***(M1)A1***

[3 marks]

d. let n be the number of balls drawn

$\mathrm{P}(X \geqslant 1) = 1 - \mathrm{P}(X = 0)$ ***M1***

$= 1 - \left(\frac{2}{3}\right)^n > 0.95$ ***M1***

$\left(\frac{2}{3}\right)^n > 0.05$

$n = 8$ ***A1***

[3 marks]

e. $8p_1 = 4.8 \Rightarrow p_1 = \frac{3}{5}$ ***(M1)A1***

$8p_2(1 - p_2) = 1.5$ ***(M1)***

$p_2^2 - p_2 - 0.1875 = 0$ ***(M1)***

$p_2 = \frac{1}{4}\ \left(\text{or } \frac{3}{4}\right)$ ***A1***

reject $\frac{3}{4}$ as it gives a total greater than one

$\mathrm{P}(1 \text{ or } 2) = \frac{17}{20}$ or $\mathrm{P}(3) = \frac{3}{20}$ ***(A1)***

recognising LCM as 20 so min total number is 20 ***(M1)***

the least possible number of 3's is 3 ***A1***

[8 marks]

Examiners report

a. Part (a) was generally well done, although many candidates lost their way after that.

b.i. Candidates had difficulty recognising all the different cases in part (b).

b.ii. Candidates had difficulty recognising all the different cases in part (b).

c. Parts (c) and (d) should have been more standard questions, but many were unable to tackle them.

d. Parts (c) and (d) should have been more standard questions, but many were unable to tackle them.

e. Part (e) was poorly answered in general.

A continuous random variable T has probability density function f defined by

$$f(t) = \begin{cases} \frac{t|\sin 2t|}{\pi}, & 0 \leqslant t \leqslant \pi \\ 0, & \text{otherwise} \end{cases}$$

a. Sketch the graph of $y = f(t)$. [2]

b. Use your sketch to find the mode of T. [1]

c. Find the mean of T. [2]

d. Find the variance of T. [3]

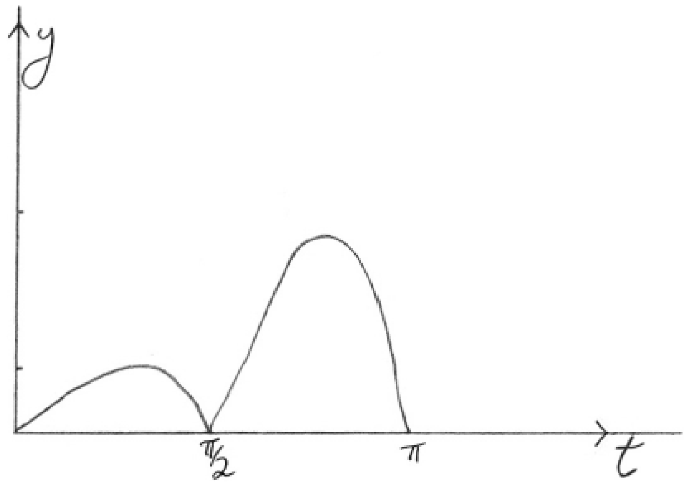
e. Find the probability that T lies between the mean and the mode. [2]

f. (i) Find $\int_0^{\pi} f(t)\mathrm{d}t$ where $0 \leqslant T \leqslant \frac{\pi}{2}$. [5]

(ii) Hence verify that the lower quartile of T is $\frac{\pi}{2}$.

Markscheme

a.



two enclosed regions ($0 \leqslant t \leqslant \frac{\pi}{2}$ and $\frac{\pi}{2} \leqslant t \leqslant \pi$) bounded by the curve and the t-axis ***A1***

correct non-symmetrical shape for $0 \leqslant t \leqslant \frac{\pi}{2}$ and

$\frac{\pi}{2} < \text{mode of } T < \pi$ clearly apparent ***A1***

[2 marks]

b. $\text{mode} = 2.46$ ***A1***

[1 mark]

c. $\mathrm{E}(T) = \frac{1}{\pi}\int_0^{\pi} t^2 \left|\sin 2t\right| \mathrm{d}t$ ***(M1)***

$= 2.04$ ***A1***

[2 marks]

d. **EITHER**

$\mathrm{Var}(T) = \int_0^{\pi} (t - 2.03788\ldots)^2 \left(\frac{t\left|\sin 2t\right|}{\pi}\right) \mathrm{d}t$ ***(M1)(A1)***

OR

$\mathrm{Var}(T) = \int_0^{\pi} t^2 \left(\frac{t\left|\sin 2t\right|}{\pi}\right) \mathrm{d}t - (2.03788\ldots)^2$

(M1)(A1)

THEN

$\mathrm{Var}(T) = 0.516$ ***A1***

[3 marks]

e. $\frac{1}{\pi}\int_{2.03788\ldots}^{2.456590\ldots} t\left|\sin 2t\right| \mathrm{d}t = 0.285$ ***(M1)A1***

[2 marks]

f. (i) attempting integration by parts ***(M1)***

$(u = t,\ \mathrm{d}u = \mathrm{d}t,\ \mathrm{d}v = \sin 2t\ \mathrm{d}t$ and $v = -\frac{1}{2}\cos 2t)$

$\frac{1}{\pi}\left[t\left(-\frac{1}{2}\cos 2t\right)\right]_0^r - \frac{1}{\pi}\int_0^r \left(-\frac{1}{2}\cos 2t\right) \mathrm{d}t$ ***A1***

Note: Award ***A1*** if the limits are not included.

$= \frac{\sin 2T}{4\pi} - \frac{T\cos 2T}{2\pi}$ ***A1***

(ii) $\frac{\sin \pi}{4\pi} - \frac{\frac{\pi}{2}\cos \pi}{2\pi} = \frac{1}{4}$ ***A1***

as $\mathrm{P}\left(0 \leqslant T \leqslant \frac{\pi}{2}\right) = \frac{1}{4}$ (or equivalent), then the lower quartile of T is $\frac{\pi}{2}$ ***R1AG***

[5 marks]

Examiners report

a. This question was generally accessible to the large majority of candidates. A substantial number of candidates were able to neatly and accurately sketch a non-symmetric bimodal continuous probability density function and to calculate its mean, mode and variance. Quite a few candidates unfortunately attempted this question with their GDC set in degrees.

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The continuous random variable *X* has probability density function f given by

$$f(x) = \begin{cases} 3ax & , \quad 0 \leqslant x < 0.5 \\ a(2-x) & , \quad 0.5 \leqslant x < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

a. Show that $a = \frac{2}{3}$. [3]

b. Find $\mathrm{P}(X < 1)$. [3]

c. Given that $\mathrm{P}\left(s < X < 0.8\right) = 2 \times \mathrm{P}\left(2s < X < 0.8\right)$, and that 0.25 < *s* < 0.4 , find the value of *s*. [7]

Markscheme

a.

$a\left[\int_0^{0.5} 3x\,\mathrm{d}x + \int_{0.5}^{2}(2-x)\,\mathrm{d}x\right] = 1$ ***M1***

Note: Award the ***M1*** for the total integral equalling 1, or equivalent.

$a\left(\frac{3}{2}\right) = 1$ ***(M1)A1***

$a = \frac{2}{3}$ ***AG***

[3 marks]

b. **EITHER**

$\int_0^{0.5} 2x\,\mathrm{d}x + \frac{2}{3}\int_{0.5}^{1}(2-x)\,\mathrm{d}x$ ***(M1)(A1)***

$= \frac{2}{3}$ ***A1***

OR

$\frac{2}{3}\int_1^2 (2-x)\,\mathrm{d}x = \frac{1}{3}$ ***(M1)***

so $\mathrm{P}\left(X < 1\right) = \frac{2}{3}$ ***(M1)A1***

[3 marks]

c. $\mathrm{P}\left(s < X < 0.8\right) = \int_s^{0.5} 2x\,\mathrm{d}x + \frac{2}{3}\int_{0.5}^{0.8}(2-x)\,\mathrm{d}x$ ***M1A1***

$= \left[x^2\right]_s^{0.5} + 0.27$

$0.25 - s^2 + 0.27$ ***(A1)***

$\mathrm{P}\left(2s < X < 0.8\right) = \frac{2}{3}\int_{2s}^{0.8}(2-x)\,\mathrm{d}x$ ***A1***

$= \frac{2}{3}\left[2x - \frac{x^2}{2}\right]_{2s}^{0.8}$

$\frac{2}{3}\left(1.28 - \left(4s - 2s^2\right)\right)$

equating

$0.25 - s^2 + 0.27 = \frac{4}{3}\left(1.28 - \left(4s - 2s^2\right)\right)$ ***(A1)***

attempt to solve for *s* ***(M1)***

s = 0.274 ***A1***

[7 marks]

Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

Students sign up at a desk for an activity during the course of an afternoon. The arrival of each student is independent of the arrival of any other student and the number of students arriving per hour can be modelled as a Poisson distribution with a mean of λ.

The desk is open for 4 hours. If exactly 5 people arrive to sign up for the activity during that time find the probability that exactly 3 of them arrived during the first hour.

Markscheme

P(3 in the first hour) $= \frac{\lambda^3 e^{-\lambda}}{3!}$ ***A1***

number to arrive in the four hours follows $Po(4\lambda)$ ***M1***

P(5 arrive in total) $= \frac{(4\lambda)^5 e^{-4\lambda}}{5!}$ ***A1***

attempt to find P(2 arrive in the next three hours) ***M1***

$= \frac{(3\lambda)^2 e^{-3\lambda}}{2!}$ ***A1***

use of conditional probability formula ***M1***

P(3 in the first hour given 5 in total) $= \frac{\frac{\lambda^3 e^{-\lambda}}{3!} \times \frac{(3\lambda)^2 e^{-3\lambda}}{2!}}{\frac{(4\lambda)^5 e^{-4\lambda}}{5!}}$ ***A1***

$\frac{\left(\frac{9}{2!3!}\right)}{\left(\frac{4^5}{5!}\right)} = \frac{45}{512} = 0.0879$ ***A1***

[8 marks]

Examiners report

A more difficult question, but it was still surprising how many candidates were unable to make a good start with it. Many were using $\lambda = \frac{5}{4}$ and consequently unable to progress very far. Many students failed to recognise that a conditional probability should be used.

Packets of biscuits are produced by a machine. The weights X, in grams, of packets of biscuits can be modelled by a normal distribution where $X \sim \mathrm{N}(\mu,\ \sigma^2)$. A packet of biscuits is considered to be underweight if it weighs less than 250 grams.

The manufacturer makes the decision that the probability that a packet is underweight should be 0.002. To do this μ is increased and σ remains unchanged.

The manufacturer is happy with the decision that the probability that a packet is underweight should be 0.002, but is unhappy with the way in which this was achieved. The machine is now adjusted to reduce σ and return μ to 253.

a. Given that $\mu = 253$ and $\sigma = 1.5$ find the probability that a randomly chosen packet of biscuits is underweight. [2]

b. Calculate the new value of μ giving your answer correct to two decimal places. [3]

c. Calculate the new value of σ. [2]

Markscheme

a. $\mathrm{P}(X < 250) = 0.0228$ ***(M1)A1***

[2 marks]

b. $\frac{250-\mu}{1.5} = -2.878\ldots$ ***(M1)(A1)***

$\Rightarrow \mu = 254.32$ ***A1***

Notes: Only award ***A1*** here if the correct 2dp answer is seen. Award ***M0*** for use of 1.5^2.

[3 marks]

c. $\frac{250-253}{\sigma} = -2.878\ldots$ ***(A1)***

$\Rightarrow \sigma = 1.04$ ***A1***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

---

a. The number of cats visiting Helena's garden each week follows a Poisson distribution with mean $\lambda = 0.6$. [9]

Find the probability that

(i) in a particular week no cats will visit Helena's garden;

(ii) in a particular week at least three cats will visit Helena's garden;

(iii) over a four-week period no more than five cats in total will visit Helena's garden;

(iv) over a twelve-week period there will be exactly four weeks in which at least one cat will visit Helena's garden.

b. A continuous random variable X has probability distribution function f given by [9]

$f(x) = k\ln x \quad 1 \leqslant x \leqslant 3$

$f(x) = 0 \quad$ otherwise

(i) Find the value of k to six decimal places.

(ii) Find the value of $\mathrm{E}(X)$.

(iii) State the mode of X.

(iv) Find the median of X.

Markscheme

a. (i) $X \sim \mathrm{Po}(0.6)$

$\mathrm{P}(X = 0) = 0.549 \ \left(= \mathrm{e}^{-0.6}\right)$ ***A1***

(ii) $\mathrm{P}(X \geqslant 3) = 1 - \mathrm{P}(X \leqslant 2)$ ***(M1)(A1)***

$= 1 - \left(\mathrm{e}^{-0.6} + \mathrm{e}^{-0.6} \times 0.6 + \mathrm{e}^{-0.6} \times \frac{0.6^2}{2}\right)$

$= 0.0231$ ***A1***

(iii) $Y \sim \mathrm{Po}(2.4)$ ***(M1)***

$\mathrm{P}(Y \leqslant 5) = 0.964$ ***A1***

(iv) $Z \sim \mathrm{B}(12, 0.451\ldots)$ ***(M1)(A1)***

Note: Award ***M1*** for recognising binomial and ***A1*** for using correct parameters.

$\mathrm{P}(Z = 4) = 0.169$ ***A1***

[9 marks]

b. (i) $k\int_1^3 \ln x\mathrm{d}x = 1$ ***(M1)***

$(k \times 1.2958\ldots = 1)$

$k = 0.771702$ ***A1***

(ii) $\mathrm{E}(X) = \int_1^3 kx\ln x\mathrm{d}x$ ***(A1)***

attempting to evaluate their integral ***(M1)***

$= 2.27$ ***A1***

(iii) $x = 3$ ***A1***

(iv) $\int_1^m k\ln x\mathrm{d}x = 0.5$ ***(M1)***

$k[x\ln x - x]_1^m = 0.5$

attempting to solve for m ***(M1)***

$m = 2.34$ ***A1***

[9 marks]

Examiners report

a. Parts (a) and (b) were generally well done by a large proportion of candidates. In part (a) (ii), some candidates used an incorrect inequality (e.g. $\mathrm{P}(X \geqslant 3) = 1 - \mathrm{P}(X \leqslant 3)$) while in (a) (iii) some candidates did not use $\mu = 2.4$. In part (a) (iv), a number of candidates either did not realise that they needed to consider a binomial random variable or did so using incorrect parameters.

b. Parts (a) and (b) were generally well done by a large proportion of candidates.

In (b) (i), some candidates gave their value of k correct to three significant figures rather than correct to six decimal places. In parts (b) (i), (ii) and (iv), a large number of candidates unnecessarily used integration by parts. In part (b) (iii), a number of candidates thought the mode of X was $f(3)$ rather than $x = 3$. In part (b) (iv), a number of candidates did not consider the domain of f when attempting to find the median or checking their solution.

In a factory producing glasses, the weights of glasses are known to have a mean of 160 grams. It is also known that the interquartile range of the weights of glasses is 28 grams. Assuming the weights of glasses to be normally distributed, find the standard deviation of the weights of glasses.

Markscheme

weight of glass = X

$X \sim \mathrm{N}(160,\ \sigma^2)$

$\mathrm{P}(X < 160 + 14) = \mathrm{P}(X < 174) = 0.75$ ***(M1)(A1)***

Note: $\mathrm{P}(X < 160 - 14) = \mathrm{P}(X < 146) = 0.25$ can also be used.

$\mathrm{P}\left(Z < \frac{14}{\sigma}\right) = 0.75$ ***(M1)***

$\frac{14}{\sigma} = 0.6745\ldots$ ***(M1)(A1)***

$\sigma = 20.8$ ***A1***

[6 marks]

Examiners report

Of those students able to start the question, there were good solutions seen. Most students could have made better use of the GDC on this question.

The age, L, in years, of a wolf can be modelled by the normal distribution $L \sim \mathrm{N}(8, 5)$.

a. Find the probability that a wolf selected at random is at least 5 years old. [2]

b. Eight wolves are independently selected at random and their ages recorded. [3]

Find the probability that more than six of these wolves are at least 5 years old.

Markscheme

a. $\mathrm{P}(L \geq 5) = 0.910$ ***(M1)A1***

[2 marks]

b. X is the number of wolves found to be at least 5 years old recognising binomial distribution ***M1***

$X \sim \mathrm{B}(8, 0.910\ldots)$

$\mathrm{P}(X > 6) = 1 - \mathrm{P}(X \leq 6)$ ***(M1)***

$= 0.843$ ***A1***

Note: Award ***M1A0*** for finding $\mathrm{P}(X \geq 6)$.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

---

a. The distance travelled by students to attend Gauss College is modelled by a normal distribution with mean 6 km and standard deviation 1.5 km. [7]

(i) Find the probability that the distance travelled to Gauss College by a randomly selected student is between 4.8 km and 7.5 km.

(ii) 15 % of students travel less than d km to attend Gauss College. Find the value of d.

b. At Euler College, the distance travelled by students to attend their school is modelled by a normal distribution with mean μ km and standard deviation σ km. [6]

If 10 % of students travel more than 8 km and 5 % of students travel less than 2 km, find the value of μ and of σ .

c. The number of telephone calls, T, received by Euler College each minute can be modelled by a Poisson distribution with a mean of 3.5. [8]

(i) Find the probability that at least three telephone calls are received by Euler College in **each** of two successive one-minute intervals.

(ii) Find the probability that Euler College receives 15 telephone calls during a randomly selected five-minute interval.

Markscheme

a. (i) $\mathrm{P}(4.8 < X < 7.5) = \mathrm{P}(-0.8 < Z < 1)$ ***(M1)***

$= 0.629$ ***A1*** ***N2***

Note: Accept $\mathrm{P}(4.8 \leqslant X \leqslant 7.5) = \mathrm{P}(-0.8 \leqslant Z \leqslant 1)$.

(ii) Stating $\mathrm{P}(X < d) = 0.15$ **or** sketching an appropriately labelled diagram. ***A1***

$\frac{d-6}{1.5} = -1.0364\ldots$ ***(M1)(A1)***

$d = (-1.0364...)(1.5) + 6$ ***(M1)***

$= 4.45$ (km) ***A1*** ***N4***

[7 marks]

b. Stating **both** $\mathrm{P}(X > 8) = 0.1$ and $\mathrm{P}(X < 2) = 0.05$ **or** sketching an appropriately labelled diagram. ***R1***

Setting up two equations in μ and σ ***(M1)***

$8 = \mu + (1.281\ldots)\sigma$ **and** $2 = \mu - (1.644\ldots)\sigma$ ***A1***

Attempting to solve for μ and σ (including by graphical means) ***(M1)***

$\sigma = 2.05$ (km) **and** $\mu = 5.37$ (km) ***A1A1*** ***N4***

Note: Accept $\mu = 5.36, 5.38$.

[6 marks]

c. (i) Use of the Poisson distribution in an inequality. ***M1***

$\mathrm{P}(T \geqslant 3) = 1 - \mathrm{P}(T \leqslant 2)$ ***(A1)***

$= 0.679...$ ***A1***

Required probability is $(0.679\ldots)^2 = 0.461$ ***M1A1*** ***N3***

Note: Allow ***FT*** for their value of $\mathrm{P}(T \geqslant 3)$.

(ii) $\tau \sim \mathrm{Po}(17.5)$ ***A1***

$\mathrm{P}(\tau = 15) = \frac{\mathrm{e}^{-17.5}(17.5)^{15}}{15!}$ ***(M1)***

$= 0.0849$ ***A1*** ***N2***

[8 marks]

Examiners report

a. This question was generally well done despite a large proportion of candidates being awarded an accuracy penalty. Candidates found part (a) (i) to be quite straightforward and was generally done very well. In part (a) (ii), a number of candidates used $\frac{d-6}{1.5}=1.0364\ldots$ instead of $\frac{d-6}{1.5}=-1.0364\ldots$. In part (b), a pleasingly high number of candidates were able to set up and solve a pair of simultaneous linear equations to correctly find the values of μ and σ. Some candidates prematurely rounded intermediate results. In part (c), a number of candidates were unable to express a correct Poisson inequality. Common errors included stating $\mathrm{P}(T\geqslant 3)=1-\mathrm{P}(T\leqslant 3)$ and using $\mu=7$.

b. This question was generally well done despite a large proportion of candidates being awarded an accuracy penalty. Candidates found part (a) (i) to be quite straightforward and was generally done very well. In part (a) (ii), a number of candidates used $\frac{d-6}{1.5}=1.0364\ldots$ instead of $\frac{d-6}{1.5}=-1.0364\ldots$. In part (b), a pleasingly high number of candidates were able to set up and solve a pair of simultaneous linear equations to correctly find the values of μ and σ. Some candidates prematurely rounded intermediate results. In part (c), a number of candidates were unable to express a correct Poisson inequality. Common errors included stating $\mathrm{P}(T\geqslant 3)=1-\mathrm{P}(T\leqslant 3)$ and using $\mu=7$.

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A random variable X has probability density function

$$f(x)=\begin{cases} ax+b, & 2\le x\le 3 \\ 0, & \text{otherwise}\end{cases}, a, b\in\mathbb{R}$$

(a) Show that $5a+2b=2$.

Let $\mathrm{E}(X)=\mu$.

(b) (i) Show that $a=12\mu-30$.

(ii) Find a similar expression for b in terms of μ.

Let the median of the distribution be 2.3.

(c) (i) Find the value of μ.

(ii) Find the value of the standard deviation of X.

Markscheme

(a) $\int_2^3 (ax+b)\mathrm{d}x\ (=1)$ ***M1A1***

$\left[\frac{1}{2}ax^2+bx\right]_2^3\ (=1)$ ***A1***

$\frac{5}{2}a+b=1$ ***M1***

$5a+2b=2$ ***AG***

[4 marks]

(b) (i) $\int_2^3 \left(ax^2+bx\right)\mathrm{d}x\ (=\mu)$ ***M1A1***

$\left[\frac{1}{3}ax^3+\frac{1}{2}bx^2\right]_2^3\ (=\mu)$ ***A1***

$\frac{19}{3}a+\frac{5}{2}b=\mu$ ***A1***

eliminating b ***M1***

eg

$\frac{19}{3}a+\frac{5}{2}\left(1-\frac{5}{2}a\right)=\mu$ ***A1***

$\frac{1}{12}a+\frac{5}{2}=\mu$

$a=12\mu-30$ ***AG***

Note: Elimination of b could be at different stages.

(ii) $b=1-\frac{5}{2}(12\mu-30)$

$=76-30\mu$ ***A1***

Note: This solution may be seen in part (i).

[7 marks]

(c) (i) $\int_2^{2.3}(ax+b)\mathrm{d}x\ (=0.5)$ ***(M1)(A1)***

$\left[\frac{1}{2}ax^2+bx\right]_2^{2.3}\ (=0.5)$

$0.645a+0.3b\ (=0.5)$ ***(A1)***

$0.645(12\mu-30)+0.3(76-30\mu)=0.5$ ***M1***

$\mu=2.34\ \left(=\frac{295}{126}\right)$ ***A1***

(ii) $\mathrm{E}\left(X^2\right)=\int_2^3 x^2(ax+b)\mathrm{d}x$ ***(M1)***

$a=12\mu-30=-\frac{40}{21},\ b=76-30\mu=\frac{121}{21}$ ***(A1)***

$\mathrm{E}\left(X^2\right)=\int_2^3 x^2\left(-\frac{40}{21}x+\frac{121}{21}\right)\mathrm{d}x=5.539\ldots\ \left(=\frac{349}{63}\right)$ ***(A1)***

$\mathrm{Var}(X)=5.539\mathrm{K}-(2.341\mathrm{K})^2=0.05813\ldots$ ***(M1)***

$\sigma=0.241$ ***A1***

[10 marks]

Total [21 marks]

Examiners report

[N/A]

A market stall sells apples, pears and plums.

a. The weights of the apples are normally distributed with a mean of 200 grams and a standard deviation of 25 grams. [5]

(i) Given that there are 450 apples on the stall, what is the expected number of apples with a weight of more than 225 grams?
(ii) Given that 70 % of the apples weigh less than m grams, find the value of m .

b. The weights of the pears are normally distributed with a mean of $\propto$ grams and a standard deviation of σ grams. Given that 8 % of these pears have a weight of more than 270 grams and 15 % have a weight less than 250 grams, find $\propto$ and σ . [6]

c. The weights of the plums are normally distributed with a mean of 80 grams and a standard deviation of 4 grams. 5 plums are chosen at random. What is the probability that exactly 3 of them weigh more than 82 grams? [3]

Markscheme

a. (i) $\mathrm{P}(X > 225) = 0.158...$ ***(M1)(A1)***

expected number $= 450 \times 0.158... = 71.4$ ***A1***

(ii) $\mathrm{P}(X < m) = 0.7$ ***(M1)***
$\Rightarrow m = 213$ (grams) ***A1***
[5 marks]

b. $\frac{270-\mu}{\sigma} = 1.40...$ ***(M1)A1***
$\frac{250-\mu}{\sigma} = -1.03...$ ***A1***
Note: These could be seen in graphical form.

solving simultaneously ***(M1)***
$\mu = 258,\ \sigma = 8.19$ ***A1A1***
[6 marks]

c. $X \sim \mathrm{N}(80, 4^2)$

$\mathrm{P}(X > 82) = 0.3085...$ ***A1***

recognition of the use of binomial distribution. ***(M1)***
$X \sim \mathrm{B}(5,\ 0.3085...)$
$\mathrm{P}(X = 3) = 0.140$ ***A1***
[3 marks]

Examiners report

a. This was an accessible question for most students with many wholly correct answers seen. In part (b) a few candidates struggled to find the correct values from the calculator and in part (c) a small minority did not see the need to treat it as a binomial distribution.

b. This was an accessible question for most students with many wholly correct answers seen. In part (b) a few candidates struggled to find the correct values from the calculator and in part (c) a small minority did not see the need to treat it as a binomial distribution.

c. This was an accessible question for most students with many wholly correct answers seen. In part (b) a few candidates struggled to find the correct values from the calculator and in part (c) a small minority did not see the need to treat it as a binomial distribution.

A ferry carries cars across a river. There is a fixed time of *T* minutes between crossings. The arrival of cars at the crossing can be assumed to follow a Poisson distribution with a mean of one car every four minutes. Let *X* denote the number of cars that arrive in *T* minutes.

a. Find *T*, to the nearest minute, if $\mathrm{P}(X \leqslant 3) = 0.6$. [3]

b. It is now decided that the time between crossings, *T*, will be 10 minutes. The ferry can carry a maximum of three cars on each trip. [4]

One day all the cars waiting at 13:00 get on the ferry. Find the probability that all the cars that arrive in the next 20 minutes will get on either the 13:10 or the 13:20 ferry.

Markscheme

a. $X \sim \mathrm{Po}(0.25\mathrm{T})$ ***(A1)***

Attempt to solve $\mathrm{P}(X \leqslant 3) = 0.6$ ***(M1)***

$T = 12.8453\ldots = 13$ (minutes) ***A1***

Note: Award ***A1M1A0*** if T found correctly but not stated to the nearest minute.

[3 marks]

b. let X_1 be the number of cars that arrive during the first interval and X_2 be the number arriving during the second.

X_1 and X_2 are Po(2.5) ***(A1)***

P (all get on) $= \mathrm{P}(X_1 \leqslant 3) \times \mathrm{P}(X_2 \leqslant 3) + \mathrm{P}(X_1 = 4) \times \mathrm{P}(X_2 \leqslant 2)$

$+\mathrm{P}(X_1 = 5) \times \mathrm{P}(X_2 \leqslant 1) + \mathrm{P}(X_1 = 6) \times \mathrm{P}(X_2 = 0)$ ***(M1)***

$= 0.573922\ldots + 0.072654\ldots + 0.019192\ldots + 0.002285\ldots$ ***(M1)***

$= 0.668\ (053\ldots)$ ***A1***

[4 marks]

Examiners report

a. There were some good answers to part (a), although poor calculator use frequently let down the candidates.

b. Very few candidates were able to access part (b).

A random variable X is normally distributed with mean μ and standard deviation σ, such that $\mathrm{P}(X < 30.31) = 0.1180$ and $\mathrm{P}(X > 42.52) = 0.3060$.

a. Find μ and σ. [6]

b. Find $\mathrm{P}\left(|X - \mu| < 1.2\sigma\right)$. [2]

Markscheme

a. $\mathrm{P}(X < 42.52) = 0.6940$ ***(M1)***

either $\mathrm{P}\left(Z < \frac{30.31-\mu}{\sigma}\right) = 0.1180$ or $\mathrm{P}\left(Z < \frac{42.52-\mu}{\sigma}\right) = 0.6940$ ***(M1)***

$\frac{30.31-\mu}{\sigma} = \underbrace{\Phi^{-1}(0.1180)}_{-1.1850\ldots}$ ***(A1)***

$\frac{42.52-\mu}{\sigma} = \underbrace{\Phi^{-1}(0.6940)}_{0.5072\ldots}$ ***(A1)***

attempting to solve simultaneously ***(M1)***

$\mu = 38.9$ and $\sigma = 7.22$ ***A1***

[6 marks]

b. $\mathrm{P}(\mu - 1.2\sigma < X < \mu + 1.2\sigma)$ (or equivalent *eg*. $2\mathrm{P}(\mu < X < \mu + 1.2\sigma)$) ***(M1)***

$= 0.770$ ***A1***

Note: Award ***(M1)A1*** for $\mathrm{P}(-1.2 < Z < 1.2) = 0.770.$

[2 marks]

Examiners report

a. [N/A]
b. [N/A]

The number of vehicles passing a particular junction can be modelled using the Poisson distribution. Vehicles pass the junction at an average rate of 300 per hour.

a. Find the probability that no vehicles pass in a given minute. [2]

b. Find the expected number of vehicles which pass in a given two minute period. [1]

c. Find the probability that more than this expected number actually pass in a given two minute period. [2]

Markscheme

a. $m = \frac{300}{60} = 5$ ***(A1)***

$\mathrm{P}(X = 0) = 0.00674$ ***A1***

or e^{-5}

[2 marks]

b. $\mathrm{E}(X) = 5 \times 2 = 10$ ***A1***

[1 mark]

c. $\mathrm{P}(X > 10) = 1 - \mathrm{P}(X \leqslant 10)$ ***(M1)***

$= 0.417$ ***A1***

[2 marks]

Examiners report

a. Parts (a) and (b) were answered successfully by many candidates. Some candidates had difficulty obtaining the correct inequality in (c).

b. Parts (a) and (b) were answered successfully by many candidates. Some candidates had difficulty obtaining the correct inequality in (c).

c. Parts (a) and (b) were answered successfully by many candidates. Some candidates had difficulty obtaining the correct inequality in (c).

The finishing times in a marathon race follow a normal distribution with mean 210 minutes and standard deviation 22 minutes.

a. Find the probability that a runner finishes the race in under three hours. [2]

b. The fastest 90% of the finishers receive a certificate. [2]

Find the time, below which a competitor has to complete the race, in order to gain a certificate.

Markscheme

a. $X \sim N(210,\ 22^2)$

$\mathrm{P}(X < 180) = 0.0863$ ***(M1)A1***

[2 marks]

b. $\mathrm{P}(X < T) = 0.9 \Rightarrow T = 238\ (\text{mins})$ ***(M1)A1***

[2 marks]

Total [5 marks]

Examiners report

a. This question was well done with many candidates obtaining full marks. On the whole, but quite a number misunderstood what was required in part (b) and 182 minutes was a repeated incorrect answer. It was disappointing that candidates have not noticed that this answer was clearly too small showing that candidates had not appreciated the context of the question.

b. This question was well done with many candidates obtaining full marks. On the whole, but quite a number misunderstood what was required in part (b) and 182 minutes was a repeated incorrect answer. It was disappointing that candidates have not noticed that this answer was clearly too small showing that candidates had not appreciated the context of the question.

The continuous random variable X has the probability distribution function $f(x) = A\sin(\ln(x)),\ 1 \le x \le 5.$

a. Find the value of A to three decimal places. [2]

b. Find the mode of X. [2]

c. Find the value $\mathrm{P}(X \le 3|X \ge 2)$. [2]

Markscheme

a. $A\int_1^5 \sin(\ln x)\mathrm{d}x = 1$ ***(M1)***

$A = 0.323\ (3\ \text{dp only})$ ***A1***

[2 marks]

b. either a graphical approach or $f'(x) = \frac{\cos(\ln x)}{x} = 0$ ***(M1)***

$x = 4.81 \quad \left(= \mathrm{e}^{\frac{\pi}{2}}\right)$ ***A1***

Note: Do not award ***A1FT*** for a candidate working in degrees.

[2 marks]

c. $\mathrm{P}(X \le 3 | X \ge 2) = \frac{\mathrm{P}(2 \le X \le 3)}{\mathrm{P}(X \ge 2)} \quad \left(= \frac{\int_2^3 \sin(\ln(x))\mathrm{d}x}{\int_2^5 \sin(\ln(x))\mathrm{d}x}\right)$ ***(M1)***

$= 0.288$ ***A1***

Note: Do not award ***A1FT*** for a candidate working in degrees.

[2 marks]

Total [6 marks]

Examiners report

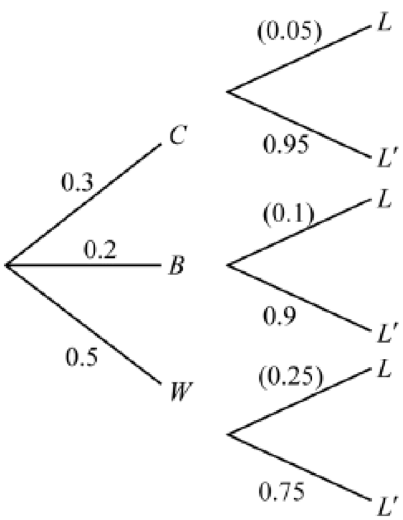
a. [N/A]
b. [N/A]
c. [N/A]

Josie has three ways of getting to school. 30% of the time she travels by car, 20% of the time she rides her bicycle and 50% of the time she walks.

When travelling by car, Josie is late 5% of the time. When riding her bicycle she is late 10% of the time. When walking she is late 25% of the time. Given that she was on time, find the probability that she rides her bicycle.

Markscheme

EITHER



M1A1A1

Note: Award ***M1*** for a two-level tree diagram, ***A1*** for correct first level probabilities, and ***A1*** for correct second level probabilities.

OR

$\mathrm{P}(B|L') = \frac{\mathrm{P}(L'|B)\mathrm{P}(B)}{\mathrm{P}(L'|B)\mathrm{P}(B)+\mathrm{P}(L'|C)\mathrm{P}(C)+\mathrm{P}(L'|W)\mathrm{P}(Q)} \quad \left(= \frac{\mathrm{P}(B \cap L')}{\mathrm{P}(L')}\right)$ ***(M1)(A1)(A1)***

THEN

$\mathrm{P}(B|L') = \frac{0.9\times0.2}{0.9\times0.2+0.95\times0.3+0.75\times0.5} \quad \left(= \frac{0.18}{0.84}\right)$ ***M1A1***

$= 0.214 \quad \left(= \frac{3}{14}\right)$ ***A1***

[6 marks]

Examiners report

[N/A]

A continuous random variable X has probability density function

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \leqslant x \leqslant 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find the probability that X lies between the mean and the mode.

Markscheme

Attempting to find the mode graphically or by using $f'(x) = 12x(2-3x)$ ***(M1)***

Mode $= \frac{2}{3}$ ***A1***

Use of $\mathrm{E}(X) = \int_0^1 xf(x)\mathrm{d}x$ ***(M1)***

$\mathrm{E}(X) = \frac{3}{5}$ ***A1***

$\int_{\frac{3}{5}}^{\frac{2}{3}} f(x)\mathrm{d}x = 0.117 \quad \left(= \frac{1981}{16\,875}\right)$ ***M1A1 N4***

[6 marks]

Examiners report

A significant number of candidates attempted to find the mode and the mean using calculus when it could be argued that these quantities could be found more efficiently with a GDC.

A significant proportion of candidates demonstrated a lack of understanding of the definitions governing the mean, mode and median of a continuous probability density function. A significant number of candidates attempted to calculate the median instead of either the mean or the mode. A number of candidates prematurely rounded their value for the mode i.e. subsequently using 0.7 for example rather than using the exact value of $\frac{2}{3}$. A few candidates offered negative probability values or probabilities greater than one.

The data of the goals scored by players in a football club during a season are given in the following table.

Goals	Frequency
0	4
1	k
2	3
3	2
4	3
8	1

a. Given that the mean number of goals scored per player is 1.95 , find the value of k. [3]

b. It is discovered that there is a mistake in the data and that the top scorer, who scored 22 goals, has not been included in the table. [3]

(i) Find the correct mean number of goals scored per player.

(ii) Find the correct standard deviation of the number of goals scored per player.

Markscheme

a. $\frac{0\bullet 4+1\bullet k+2\bullet 3+3\bullet 2+4\bullet 3+8\bullet 1}{13+k}=1.95$ $\left(\frac{k+32}{k+13}=1.95\right)$ ***(M1)***

attempting to solve for k ***(M1)***

$k=7$ ***A1***

[3 marks]

b. (i) $\frac{7+32+22}{7+13+1}=2.90$ $\left(=\frac{61}{21}\right)$ ***(M1)A1***

(ii) standard deviation $=4.66$ ***A1***

Note: Award ***A0*** for 4.77.

[3 marks]

Total [6 marks]

Examiners report

a. [N/A]
b. [N/A]

The weights, in kg, of male birds of a certain species are modelled by a normal distribution with mean μ and standard deviation σ .

a. Given that 70 % of the birds weigh more than 2.1 kg and 25 % of the birds weigh more than 2.5 kg, calculate the value of μ and the value of σ . [4]

b. A random sample of ten of these birds is obtained. Let X denote the number of birds in the sample weighing more than 2.5 kg. [5]

(i) Calculate $\mathrm{E}(X)$.

(ii) Calculate the probability that exactly five of these birds weigh more than 2.5 kg.

(iii) Determine the most likely value of X.

c. The number of eggs, Y, laid by female birds of this species during the nesting season is modelled by a Poisson distribution with mean λ. [8]

You are given that $\mathrm{P}(Y \geqslant 2) = 0.80085$, correct to 5 decimal places.

(i) Determine the value of λ.

(ii) Calculate the probability that two randomly chosen birds lay a total of two eggs between them.

(iii) Given that the two birds lay a total of two eggs between them, calculate the probability that they each lay one egg.

Markscheme

a. we are given that

$2.1 = \mu - 0.5244\sigma$

$2.5 = \mu + 0.6745\sigma$ ***M1A1***

$\mu = 2.27$, $\sigma = 0.334$ ***A1A1***

[4 marks]

b. (i) let X denote the number of birds weighing more than 2.5 kg

then X is B(10, 0.25) ***A1***

$\mathrm{E}(X) = 2.5$ ***A1***

(ii) 0.0584 ***A1***

(iii) to find the most likely value of X, consider

$p_0 = 0.0563\ldots,\ p_1 = 0.1877\ldots,\ p_2 = 0.2815\ldots,\ p_3 = 0.2502\ldots$ ***M1***

therefore, most likely value = 2 ***A1***

[5 marks]

c. (i) we solve $1 - \mathrm{P}(Y \leqslant 1) = 0.80085$ using the GDC ***M1***

$\lambda = 3.00$ ***A1***

(ii) let $X_1,\ X_2$ denote the number of eggs laid by each bird

$\mathrm{P}(X_1 + X_2 = 2) = \mathrm{P}(X_1 = 0)\mathrm{P}(X_2 = 1) + \mathrm{P}(X_1 = 1)\mathrm{P}(X_2 = 1) + \mathrm{P}(X_1 = 2)\mathrm{P}(X_2 = 0)$ ***M1A1***

$= \mathrm{e}^{-3} \times \mathrm{e}^{-3} \times \frac{9}{2} + (\mathrm{e}^{-3} \times 3)^2 + \mathrm{e}^{-3} \times \frac{9}{2} \times \mathrm{e}^{-3} = 0.0446$ ***A1***

(iii) $\mathrm{P}(X_1 = 1,\ X_2 = 1 | X_1 + X_2 = 2) = \frac{\mathrm{P}(X_1=1,\ X_2=1)}{\mathrm{P}(X_1+X_2=2)}$ ***M1A1***

$= 0.5$ ***A1***

[8 marks]

Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

A fisherman notices that in any hour of fishing, he is equally likely to catch exactly two fish, as he is to catch less than two fish. Assuming the number of fish caught can be modelled by a Poisson distribution, calculate the expected value of the number of fish caught when he spends four hours fishing.

Markscheme

$X \sim \mathrm{Po}(m)$

$\mathrm{P}(X = 2) = \mathrm{P}(X < 2)$ ***(M1)***

$\frac{1}{2}m^2\mathrm{e}^{-m} = \mathrm{e}^{-m}(1 + m)$ ***(A1)(A1)***

$m = 2.73 \ \left(1 + \sqrt{3}\right)$ ***A1***

in four hours the expected value is $10.9 \ \left(4 + 4\sqrt{3}\right)$ ***A1***
Note: Value of *m* does not need to be rounded.

[5 marks]

Examiners report

Many candidates did not attempt this question and many others did not go beyond setting the equation up. Among the ones who attempted to solve the equation, once again, very few candidates took real advantage of GDC use to obtain the correct answer.

A student arrives at a school X minutes after 08:00, where X may be assumed to be normally distributed. On a particular day it is observed that 40% of the students arrive before 08:30 and 90% arrive before 08:55.

Consider the function $f(x) = \frac{\ln x}{x}$, $0 < x < \mathrm{e}^2$.

a. Find the mean and standard deviation of X. [5]

b. The school has 1200 students and classes start at 09:00. Estimate the number of students who will be late on that day. [3]

c. Maelis had not arrived by 08:30. Find the probability that she arrived late. [2]

d. At 15:00 it is the end of the school day and it is assumed that the departure of the students from school can be modelled by a Poisson distribution. On average 24 students leave the school every minute. [3]

Find the probability that at least 700 students leave school before 15:30.

e. At 15:00 it is the end of the school day and it is assumed that the departure of the students from school can be modelled by a Poisson distribution. On average 24 students leave the school every minute. [4]

There are 200 days in a school year. Given that Y denotes the number of days in the year that at least 700 students leave before 15:30, find

(i) $\mathrm{E}(Y)$;

(ii) $P(Y > 150)$.

Markscheme

a. $\mathrm{P}(X < 30) = 0.4$

$\mathrm{P}(X < 55) = 0.9$

or relevant sketch ***(M1)***

given $Z = \frac{X-\mu}{\sigma}$

$\mathrm{P}(Z < z) = 0.4 \Rightarrow \frac{30-\mu}{\sigma} = -0.253...$ ***(A1)***

$\mathrm{P}(Z < z) = 0.9 \Rightarrow \frac{55-\mu}{\sigma} = 1.28...$ ***(A1)***

$\mu = 30 + (0.253...) \times \sigma = 55 - (1.28...) \times \sigma$ ***M1***

$\sigma = 16.3$, $\mu = 34.1$ ***A1***

Note: Accept 16 and 34.

Note: Working with 830 and 855 will only gain the two ***M*** marks.

[5 marks]

b. X ~ N(34.12 . . ., 16.28...^{2})

late to school $\Rightarrow X > 60$

$\mathrm{P}(X > 60) = 0.056...$ ***(A1)***

number of students late $=$ $0.0560...\times 1200$ ***(M1)***

$= 67$ (to nearest integer) A1

Note: Accept 62 for use of 34 and 16.

[3 marks]

c. $\mathrm{P}(X > 60|X > 30) = \frac{\mathrm{P}(X>60)}{\mathrm{P}(X>30)}$ ***M1***

$= 0.0935$ (accept anything between 0.093 and 0.094) ***A1***

Note: If 34 and 16 are used 0.0870 is obtained. This should be accepted.

[2 marks]

d. let L be the random variable of the number of students who leave school in a 30 minute interval

since $24 \times 30 = 720$ ***A1***

L ~ $\mathrm{Po}(720)$

$\mathrm{P}(L \geqslant 700) = 1 - \mathrm{P}(L \leqslant 699)$ ***(M1)***

$= 0.777$ ***A1***

Note: Award ***M1A0*** for $P(L > 700) = 1 - P(L \leqslant 700)$ (this leads to 0.765).

[3 marks]

e. (i) $Y \sim \mathrm{B}(200, 0.7767\ldots)$ ***(M1)***

$\mathrm{E}(Y) = 200 \times 0.7767\ldots = 155$ ***A1***

Note: On ft, use of 0.765 will lead to 153.

(ii) $\mathrm{P}(Y > 150) = 1 - \mathrm{P}(Y \leqslant 150)$ ***(M1)***

$= 0.797$ ***A1***

Note: Accept 0.799 from using rounded answer.

Note: On ft, use of 0.765 will lead to 0.666.

[4 marks]

Examiners report

a. Candidates who had been prepared to solve questions from this part of the syllabus did well on the question. As a general point, candidates did not always write down clearly which distribution was being used. There were many candidates who seemed unfamiliar with the concept of Normal distributions as well as the Poisson and Binomial distributions and did not attempt the question. Parts (a) – (c) of the question were a variation on similar problems seen on previous examinations and there were a disappointing number of candidates who seemed unable to start the question. The use of 830 and 850 rather than minutes after 8am was seen and this caused students to lose marks despite knowing the method required. In general technology was used well and this was seen in (d) when solving a problem that involved a Poisson distribution. A number of candidates were unable to identify the Binomial distribution in (e).

b. Candidates who had been prepared to solve questions from this part of the syllabus did well on the question. As a general point, candidates did not always write down clearly which distribution was being used. There were many candidates who seemed unfamiliar with the concept of Normal distributions as well as the Poisson and Binomial distributions and did not attempt the question. Parts (a) – (c) of the question were a variation on similar problems seen on previous examinations and there were a disappointing number of candidates who seemed unable to start the question. The use of 830 and 850 rather than minutes after 8am was seen and this caused students to lose marks despite knowing the method required. In general technology was used well and this was seen in (d) when solving a problem that involved a Poisson distribution. A number of candidates were unable to identify the Binomial distribution in (e).

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A mosaic is going to be created by randomly selecting 1000 small tiles, each of which is either black or white. The probability that a tile is white is 0.1. Let the random variable W be the number of white tiles.

a. State the distribution of W, including the values of any parameters. [2]

b. Write down the mean of W. [1]

c. Find $\mathrm{P}(W > 89)$. [2]

Markscheme

a. $W \sim B(1000,\ 0.1)$ $\left(\text{accept } C_k^{1000}(0.1)^k(0.9)^{1000-k}\right)$ ***A1A1***

Note: First ***A1*** is for recognizing the binomial, second ***A1*** for both parameters if stated explicitly in this part of the question.

[2 marks]

b. $\mu(= 1000 \times 0.1) = 100$ ***A1***

[1 mark]

c. $\mathrm{P}(W > 89) = \mathrm{P}(W \geq 90)$ $(= 1 - \mathrm{P}(W \leq 89))$ ***(M1)***

$= 0.867$ ***A1***

Notes: Award ***M0A0*** for 0.889

[2 marks]

Total [5 marks]

Examiners report

a. Overall this question was well answered. In part (a) a number of candidates did not mention the binomial distribution or failed to state its parameters although they could go on and do the next parts.

b. In part (b) most candidates could state the expected value.

c. In part (c) many candidates had problems with inequalities due to the discrete nature of the variable. Most candidates that could deal with the inequality were able to use the GDC to obtain the answer.

Ava and Barry play a game with a bag containing one green marble and two red marbles. Each player in turn randomly selects a marble from the bag, notes its colour and replaces it. Ava wins the game if she selects a green marble. Barry wins the game if he selects a red marble. Ava starts the game.

a. Find the probability that Ava wins on her first turn. [1]

b. Find the probability that Barry wins on his first turn. [2]

c. Find the probability that Ava wins in one of her first three turns. [4]

d. Find the probability that Ava eventually wins. [4]

Markscheme

a. $\mathrm{P}(\text{Ava wins on her first turn}) = \frac{1}{3}$ ***A1***

[1 mark]

b. $\mathrm{P}(\text{Barry wins on his first turn}) = \left(\frac{2}{3}\right)^2$ ***(M1)***

$= \frac{4}{9} \quad (= 0.444)$ ***A1***

[2 marks]

c. P(Ava wins in one of her first three turns)

$= \frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3}$ ***M1A1A1***

Note: Award ***M1*** for adding probabilities, award ***A1*** for a correct second term and award ***A1*** for a correct third term.

Accept a correctly labelled tree diagram, awarding marks as above.

$= \frac{103}{243} \quad (= 0.424)$ ***A1***

[4 marks]

d. $\mathrm{P(Ava\ eventually\ wins)} = \frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \ldots$ ***(A1)***

using $S_\infty = \frac{a}{1-r}$ with $a = \frac{1}{3}$ and $r = \frac{2}{9}$ ***(M1)(A1)***

Note: Award ***(M1)*** for using $S_\infty = \frac{a}{1-r}$ and award ***(A1)*** for $a = \frac{1}{3}$ and $r = \frac{2}{9}$.

$= \frac{3}{7} \quad (= 0.429)$ ***A1***

[4 marks]

Total [11 marks]

Examiners report

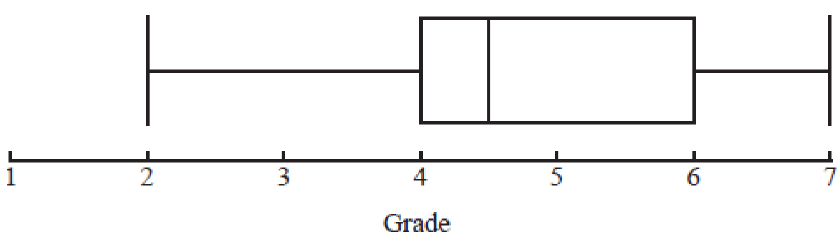
a. Parts (a) and (b) were straightforward and were well done.

b. Parts (a) and (b) were straightforward and were well done.

c. Parts (c) and (d) were also reasonably well done.

d. Parts (c) and (d) were also reasonably well done. A pleasingly large number of candidates recognized that an infinite geometric series was required in part (d).

The box and whisker plot below illustrates the IB grades obtained by 100 students.



IB grades can only take integer values.

a. How many students obtained a grade of more than 4? [1]

b. State, with reasons, the maximum possible number and minimum possible number of students who obtained a 4 in the exam. [4]

Markscheme

a. 50 ***A1***

[1 mark]

b. Lower quartile is 4 so at least 26 obtained a 4 ***R1***

Lower bound is 26 ***A1***
Minimum is 2 but the rest could be 4 ***R1***
So upper bound is 49 ***A1***

Note: Do not allow follow through for ***A*** marks.

Note: If answers are incorrect award ***R0A0***; if argument is correct but no clear lower/upper bound is stated award ***R1A0;*** award ***R0A1*** for correct answer without explanation or incorrect explanation.

[4 marks]

Examiners report

a. Very few candidates were successful in answering this question. In many cases it was clear that candidates were not familiar with box-and-whisker plots at all; in other cases the explanations given revealed various misconceptions.

b. Very few candidates were successful in answering this question. In many cases it was clear that candidates were not familiar with box-and-whisker plots at all; in other cases the explanations given revealed various misconceptions.

Six customers wait in a queue in a supermarket. A customer can choose to pay with cash or a credit card. Assume that whether or not a customer pays with a credit card is independent of any other customers' methods of payment.

It is known that 60% of customers choose to pay with a credit card.

(a) Find the probability that:

(i) the first three customers pay with a credit card and the next three pay with cash;

(ii) exactly three of the six customers pay with a credit card.

There are *n* customers waiting in another queue in the same supermarket. The probability that at least one customer pays with cash is greater than 0.995.

(b) Find the minimum value of *n*.

Markscheme

(a) (i) $0.6^3 \times 0.4^3$ ***(M1)***

Note: Award ***(M1)*** for use of the product of probabilities.

$= 0.0138$ ***A1***

(ii) binomial distribution $X : \mathrm{B}(6, 0.6)$ ***(M1)***

Note: Award ***(M1)*** for recognizing the binomial distribution.

$\mathrm{P}(X = 3) = {}^6C_3(0.6)^3(0.4)^3$

$= 0.276$ ***A1***

Note: Award ***(M1)A1*** for ${}^6C_3 \times 0.0138 = 0.276$.

[4 marks]

(b) $Y : \mathrm{B}(n,\ 0.4)$

$\mathrm{P}(Y \geqslant 1) > 0.995$

$1 - \mathrm{P}(Y = 0) > 0.995$
$\mathrm{P}(Y = 0) < 0.005$ ***(M1)***

Note: Award ***(M1)*** for any of the last three lines. Accept equalities.

$0.6^n < 0.005$ ***(M1)***

Note: Award ***(M1)*** for attempting to solve $0.6^n < 0.005$ using any method, *eg*, logs, graphically, use of solver. Accept an equality.

$n > 10.4$
$\therefore n = 11$ ***A1***
[3 marks]

Total [7 marks]

Examiners report

[N/A]

Jan and Sia have been selected to represent their country at an international discus throwing competition. Assume that the distance thrown by each athlete is normally distributed. The mean distance thrown by Jan in the past year was 60.33 metres with a standard deviation of 1.95 metres.

a. In the past year, 80 % of Jan's throws have been longer than *x* metres. Find *x* correct to two decimal places. [2]

b. In the past year, 80 % of Sia's throws have been longer than 56.52 metres. If the mean distance of her throws was 59.39 metres, find the standard deviation of her throws. [3]

c. This year, Sia's throws have a mean of 59.50 metres and a standard deviation of 3.00 metres. The mean and standard deviation of Jan's throws have remained the same. In the competition, an athlete must have at least one throw of 65 metres or more in the first round to qualify for the final round. Each athlete is allowed three throws in the first round. [10]

(i) Determine whether Jan or Sia is more likely to qualify for the final on their first throw.

(ii) Find the probability that both athletes qualify for the final.

Markscheme

a. $X \sim \mathrm{N}(60.33,\, 1.95^2)$

$\mathrm{P}(X < x) = 0.2 \Rightarrow x = 58.69\ \mathrm{m}$ ***(M1)A1***

[2 marks]

b. $z = -0.8416\ldots$ ***(A1)***

$-0.8416 = \frac{56.52 - 59.39}{\sigma}$ ***(M1)***

$\sigma \approx 3.41$ ***A1***

[3 marks]

c. Jan $X \sim \mathrm{N}(60.33,\, 1.95^2)$; Sia $X \sim \mathrm{N}(59.50,\, 3.00^2)$

(i) Jan: $\mathrm{P}(X > 65) \approx 0.00831$ ***(M1)A1***

Sia: $\mathrm{P}(Y > 65) \approx 0.0334$ ***A1***

Sia is more likely to qualify ***R1***

Note: Only award ***R1*** if ***(M1)*** has been awarded.

(ii) Jan: $\mathrm{P}(X \geqslant 1) = 1 - \mathrm{P}(X = 0)$ ***(M1)***

$= 1 - (1 - 0.00831\ldots)^3 \approx 0.0247$ ***(M1)A1***

Sia: $\mathrm{P}(Y \geqslant 1) = 1 - \mathrm{P}(Y = 0) = 1 - (1 - 0.0334\ldots)^3 \approx 0.0968$ ***A1***

Note: Accept 0.0240 and 0.0969.

hence, $\mathrm{P}(X \geqslant 1 \text{ and } y \geqslant 1) = 0.0247 \times 0.0968 = 0.00239$ ***(M1)A1***

[10 marks]

Examiners report

a. Parts (a) and (b) were generally accessible to many candidates. In (c)(i) quite a few candidates missed the wording ‘first throw’, and consequently in (ii) used the incorrect probabilities.

b. Parts (a) and (b) were generally accessible to many candidates. In (c)(i) quite a few candidates missed the wording ‘first throw’, and consequently in (ii) used the incorrect probabilities.

c. Parts (a) and (b) were generally accessible to many candidates. In (c)(i) quite a few candidates missed the wording ‘first throw’, and consequently in (ii) used the incorrect probabilities.

The times taken for male runners to complete a marathon can be modelled by a normal distribution with a mean 196 minutes and a standard deviation 24 minutes.

It is found that 5% of the male runners complete the marathon in less than T_1 minutes.

The times taken for female runners to complete the marathon can be modelled by a normal distribution with a mean 210 minutes. It is found that 58% of female runners complete the marathon between 185 and 235 minutes.

a. Find the probability that a runner selected at random will complete the marathon in less than 3 hours. [2]

b. Calculate T_1. [2]

c. Find the standard deviation of the times taken by female runners. [4]

Markscheme

a. $T \sim N(196,\ 24^2)$

$\mathrm{P}(T < 180) = 0.252$ ***(M1)A1***

[2 marks]

b. $\mathrm{P}(T < T_1) = 0.05$ ***(M1)***

$T_1 = 157$ ***A1***

[2 marks]

c. $F \sim N(210,\ \sigma^2)$

$\mathrm{P}(F < 235) = 0.79$ ***(M1)***

$\frac{235-210}{\sigma} = 0.806421$ or equivalent ***(M1)(A1)***

$\sigma = 31.0$ ***A1***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

(a) Ahmed is typing Section A of a mathematics examination paper. The number of mistakes that he makes, X, can be modelled by a Poisson distribution with mean 3.2 . Find the probability that Ahmed makes exactly four mistakes.

(b) His colleague, Levi, is typing Section B of this paper. The number of mistakes that he makes, Y, can be modelled by a Poisson distribution with mean m.

(i) If $\mathrm{E}(Y^2) = 5.5$, find the value of m.

(ii) Find the probability that Levi makes exactly three mistakes.

(c) Given that X and Y are independent, find the probability that Ahmed makes exactly four mistakes and Levi makes exactly three mistakes.

Markscheme

(a) $X \sim \mathrm{Po}(3.2)$

$\mathrm{P}(X = 4) = \frac{\mathrm{e}^{-3.2}3.2^4}{4!}$

$= 0.178$ ***A1***

(b) (i) $\mathrm{Var}(Y) = \mathrm{E}(Y^2) - \mathrm{E}^2(Y)$ ***(M1)***

$m = 5.5 - m^2$ ***A1***

$m = 1.90$ (or $m = -2.90$ which is valid) ***A1***

(ii) $Y \sim \text{Po}(1.90)$

$\text{P}(Y = 3) = \frac{\text{e}^{-1.90} 1.90^4}{3!}$ ***(M1)***

$= 0.171$ ***A1***

(c) Required probability $= 0.171 \times 0.178 = 0.0304$ (accept 0.0305) ***(M1)A1***

[8 marks]

Examiners report

Part (a) was correctly solved by most candidates, either using the formula or directly from their GDC. Solutions to (b), however, were extremely disappointing with the majority of candidates giving $\sqrt{5}$, incorrectly, as their value of *m* . It was possible to apply follow through in (b) (ii) and (c) which were well done in general.

A survey is conducted in a large office building. It is found that 30% of the office workers weigh less than 62 kg and that 25% of the office workers weigh more than 98 kg.

The weights of the office workers may be modelled by a normal distribution with mean μ and standard deviation σ.

a. (i) Determine two simultaneous linear equations satisfied by μ and σ. [6]

(ii) Find the values of μ and σ.

b. Find the probability that an office worker weighs more than 100 kg. [1]

c. There are elevators in the office building that take the office workers to their offices. [2]

Given that there are 10 workers in a particular elevator,

find the probability that at least four of the workers weigh more than 100 kg.

d. Given that there are 10 workers in an elevator and at least one weighs more than 100 kg, [3]

find the probability that there are fewer than four workers exceeding 100 kg.

e. The arrival of the elevators at the ground floor between $08:00$ and $09:00$ can be modelled by a Poisson distribution. Elevators arrive on average every 36 seconds. [3]

Find the probability that in any half hour period between $08:00$ and $09:00$ more than 60 elevators arrive at the ground floor.

f. An elevator can take a maximum of 10 workers. Given that 400 workers arrive in a half hour period independently of each other, [3]

find the probability that there are sufficient elevators to take them to their offices.

Markscheme

a. **Note:** In Section B, accept answers that correctly round to 2 sf.

(i) let W be the weight of a worker and $W \sim \text{N}(\mu,\ \sigma^2)$

$\mathrm{P}\left(Z < \frac{62-\mu}{\alpha}\right) = 0.3$ and $\mathrm{P}\left(Z < \frac{98-\mu}{\sigma}\right) = 0.75$ ***(M1)***

Note: Award ***M1*** for a correctly shaded and labelled diagram.

$\frac{62-\mu}{\sigma} = \Phi^{-1}(0.3) \quad (= -0.524\ldots)$ and
$\frac{98-\mu}{\sigma} = \Phi^{-1}(0.75) \quad (= 0.674\ldots)$
or linear equivalents ***A1A1***

Note: Condone equations containing the GDC inverse normal command.

(ii) attempting to solve simultaneously ***(M1)***

$\mu = 77.7,\ \sigma = 30.0$ ***A1A1***

[6 marks]

b. **Note:** In Section B, accept answers that correctly round to 2 sf.

$\mathrm{P}(W > 100) = 0.229$ ***A1***

[1 mark]

c. **Note:** In Section B, accept answers that correctly round to 2 sf.

let X represent the number of workers over 100 kg in a lift of ten passengers

$X \sim \mathrm{B}(10,\ 0.229\ldots)$ ***(M1)***

$\mathrm{P}(X \geq 4) = 0.178$ ***A1***

[2 marks]

d. **Note:** In Section B, accept answers that correctly round to 2 sf.

$\mathrm{P}(X < 4 | X \geq 1) = \frac{\mathrm{P}(1 \leq X \leq 3)}{\mathrm{P}(X \geq 1)}$ ***M1(A1)***

Note: Award the ***M1*** for a clear indication of a conditional probability.

$= 0.808$ ***A1***

[3 marks]

e. **Note:** In Section B, accept answers that correctly round to 2 sf.

$L \sim \mathrm{Po}(50)$ ***(M1)***

$\mathrm{P}(L > 60) = 1 - \mathrm{P}(L \leq 60)$ ***(M1)***

$= 0.0722$ ***A1***

[3 marks]

f. **Note:** In Section B, accept answers that correctly round to 2 sf.

400 workers require at least 40 elevators ***(A1)***

$\mathrm{P}(L \geq 40) = 1 - \mathrm{P}(L \leq 39)$ ***(M1)***

$= 0.935$ ***A1***

[3 marks]

Total [18 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

---

a. Farmer Suzie grows turnips and the weights of her turnips are normally distributed with a mean of $122g$ and standard deviation of $14.7g$. [6]

(i) Calculate the percentage of Suzie's turnips that weigh between $110g$ and $130g$.

(ii) Suzie has 100 turnips to take to market. Find the expected number weighing more than $130g$.

(iii) Find the probability that at least 30 of the $100g$ turnips weigh more than $130g$.

b. Farmer Ray also grows turnips and the weights of his turnips are normally distributed with a mean of $144g$. Ray only takes to market turnips that weigh more than $130g$. Over a period of time, Ray finds he has to reject 1 in 15 turnips due to their being underweight. [6]

(i) Find the standard deviation of the weights of Ray's turnips.

(ii) Ray has 200 turnips to take to market. Find the expected number weighing more than $150g$.

Markscheme

a. (i) $P(110 < X < 130) = 0.49969\ldots = 0.500 = 50.0\%$ ***(M1)A1***

Note: Accept 50

Note: Award ***M1A0*** for 0.50 (0.500)

(ii) $P(X > 130) = (1 - 0.707\ldots) = 0.293\ldots$ ***M1***

expected number of turnips $= 29.3$ ***A1***

Note: Accept 29.

(iii) no of turnips weighing more than 130 is $Y \sim B(100,\ 0.293)$ ***M1***

$P(Y \geq 30) = 0.478$ ***A1***

[6 marks]

b. (i) $X \sim N(144,\ \sigma^2)$

$P(X \leq 130) = \frac{1}{15} = 0.0667$ ***(M1)***

$P\left(Z \leq \frac{130-144}{\sigma}\right) = 0.0667$

$\frac{14}{\sigma} = 1.501$ ***(A1)***

$\sigma = 9.33\ \mathrm{g}$ ***A1***

(ii) $P(X > 150 | X > 130) = \frac{P(X>150)}{P(X>130)}$ ***M1***

$= \frac{0.26008\ldots}{1-0.06667} = 0.279$ ***A1***

expected number of turnips $= 55.7$ ***A1***

[6 marks]

Total [12 marks]

Examiners report

a. [N/A]
b. [N/A]

Emma acquires a new cell phone for her birthday and receives texts from her friends. It is assumed that the daily number of texts Emma receives follows a Poisson distribution with mean $m = 5$.

a. (i) Find the probability that on a certain day Emma receives more than 7 texts. [4]

(ii) Determine the expected number of days in a week on which Emma receives more than 7 texts.

b. Find the probability that Emma receives fewer than 30 texts during a week. [3]

Markscheme

a. (i) $X \sim Po(5)$

$\mathrm{P}(X \geq 8) = 0.133$ ***(M1)A1***

(ii) $7 \times 0.133\ldots$ ***M1***

$\approx 0.934\ \text{days}$ ***A1***

Note: Accept “1 day”.

[4 marks]

b. $7 \times 5 = 35 \quad (Y \sim Po(35))$ ***(A1)***

$\mathrm{P}(Y \leq 29) = 0.177$ ***(M1)A1***

[3 marks]

Total [7 marks]

Examiners report

a. [N/A]
b. [N/A]

The number of complaints per day received by customer service at a department store follows a Poisson distribution with a mean of 0.6.

a. On a randomly chosen day, find the probability that [3]

(i) there are no complaints;

(ii) there are at least three complaints.

b. In a randomly chosen five-day week, find the probability that there are no complaints. [2]

c. On a randomly chosen day, find the most likely number of complaints received. [3]

Justify your answer.

d. The department store introduces a new policy to improve customer service. The number of complaints received per day now follows a Poisson distribution with mean λ. [2]

On a randomly chosen day, the probability that there are no complaints is now 0.8.

Find the value of λ.

Markscheme

a. (i) $P(X = 0) = 0.549\ (= \mathrm{e}^{-0.6})$ ***A1***

(ii) $P(X \geq 3) = 1 - P(X \leq 2)$ ***(M1)***

$P(X \geq 3) = 0.0231$ ***A1***

[3 marks]

b. **EITHER**

using $Y \sim \mathrm{Po}(3)$ ***(M1)***

OR

using $(0.549)^5$ ***(M1)***

THEN

$\mathrm{P}(Y = 0) = 0.0498\ \left(= \mathrm{e}^{-3}\right)$ ***A1***

[2 marks]

c. $\mathrm{P}(X = 0)$ (most likely number of complaints received is zero) ***A1***

EITHER

calculating $\mathrm{P}(X = 0) = 0.549$ and $\mathrm{P}(X = 1) = 0.329$ ***M1A1***

OR

sketching an appropriate (discrete) graph of $\mathrm{P}(X = x)$ against x ***M1A1***

OR

finding $\mathrm{P}(X = 0) = e^{-0.6}$ and stating that $\mathrm{P}(X = 0) > 0.5$ ***M1A1***

OR

using $\mathrm{P}(X = x) = \mathrm{P}(X = x - 1) \times \frac{\mu}{x}$ where $\mu < 1$ ***M1A1***

[3 marks]

d. $\mathrm{P}(X = 0) = 0.8 \ (\Rightarrow e^{-\lambda} = 0.8)$ ***(A1)***

$\lambda = 0.223 \left(= \ln \frac{5}{4}, = -\ln \frac{4}{5}\right)$ ***A1***

[2 marks]

Total [10 marks]

Examiners report

a. Parts (a), (b) and (d) were generally well done. In (a) (ii), some candidates calculated $1 - \mathrm{P}(X \le 3)$.

b. Parts (a), (b) and (d) were generally well done.

c. A number of candidates offered clear and well-reasoned solutions to part (c). The two most common successful approaches used to justify that the most likely number of complaints received is zero were either to calculate $\mathrm{P}(X = x)$ for $x = 0,\ 1,\ \ldots$ or find that $\mathrm{P}(X = 0) = 0.549\ (> 0.5)$. A number of candidates stated that the most number of complaints received was the mean of the distribution $(\lambda = 0.6)$.

d. Parts (a), (b) and (d) were generally well done.

The random variable X follows a Poisson distribution with mean $m \neq 0$.

a. Given that $2\mathrm{P}(X = 4) = \mathrm{P}(X = 5)$, show that $m = 10$. [3]

b. Given that $X \le 11$, find the probability that $X = 6$. [4]

Markscheme

a. $2\frac{\mathrm{e}^{-m}m^4}{4!} = \frac{\mathrm{e}^{-m}m^5}{5!}$ ***M1A1***

$\frac{2}{4!} = \frac{m}{5!}$ or other simplification ***M1***

Note: accept a labelled graph showing clearly the solution to the equation. Do not accept simple verification that $m = 10$ is a solution.

$\Rightarrow m = 10$ ***AG***

[3 marks]

b. $\mathrm{P}(X = 6|X \leq 11) = \frac{\mathrm{P}(X=6)}{\mathrm{P}(X\leq 11)}$ ***(M1) (A1)***

$= \frac{0.063055\ldots}{0.696776\ldots}$ ***(A1)***

$= 0.0905$ ***A1***

[4 marks]

Total [7 marks]

Examiners report

a. Most candidates successfully finished part (a) with two fundamental errors occurring regularly. Either e was granted powers of 4 and 5 or an attempt to show that the value of m was 10 was made by evaluation.

b. Part (b) was challenging for many candidates that showed that the idea of conditional probability was poorly understood. There were many incorrect solutions where often candidates only found $\mathrm{P}(X = 6)$.

It is given that one in five cups of coffee contain more than 120 mg of caffeine.

It is also known that three in five cups contain more than 110 mg of caffeine.

Assume that the caffeine content of coffee is modelled by a normal distribution.
Find the mean and standard deviation of the caffeine content of coffee.

Markscheme

let X be the random variable “amount of caffeine content in coffee”

$\mathrm{P}(X > 120) = 0.2,\ \mathrm{P}(X > 110) = 0.6$ ***(M1)***

$(\Rightarrow \mathrm{P}(X < 120) = 0.8,\ \mathrm{P}(X < 110) = 0.4)$

Note: Award ***M1*** for at least one correct probability statement.

$\frac{120-\mu}{\sigma} = 0.84162\ldots,\ \frac{110-\mu}{\sigma} = -0.253347\ldots$ ***(M1)(A1)(A1)***

Note: Award ***M1*** for attempt to find at least one appropriate z-value.

$120 - \mu = 0.84162\sigma,\ 110 - \mu = -0.253347\sigma$

attempt to solve simultaneous equations ***(M1)***

$\mu = 112,\ \sigma = 9.13$ ***A1***

[6 marks]

Examiners report

[N/A]

A team of 6 players is to be selected from 10 volleyball players, of whom 8 are boys and 2 are girls.

a. In how many ways can the team be selected? [2]

b. In how many of these selections is exactly one girl in the team? [3]

c. If the selection of the team is made at random, find the probability that exactly one girl is in the team. [2]

Markscheme

a. $\binom{10}{6} = 210$ ***(M1)A1***

[2 marks]

b. $2 \times \binom{8}{5} = 112$ ***(M1)A1A1***

Note: Accept $210 - 28 - 70 = 112$

[3 marks]

c. $\frac{112}{210}\left(= \frac{8}{15} = 0.533\right)$ ***(M1)A1***

[2 marks]

Examiners report

a. Most candidates answered this question well although in some cases candidates were not able to distinguish the use of permutations from combinations. Almost all candidates scored the two marks of part (c), but many of these were follow through marks.

b. Most candidates answered this question well although in some cases candidates were not able to distinguish the use of permutations from combinations. Almost all candidates scored the two marks of part (c), but many of these were follow through marks.

c. Most candidates answered this question well although in some cases candidates were not able to distinguish the use of permutations from combinations. Almost all candidates scored the two marks of part (c), but many of these were follow through marks.

A random variable X has a probability distribution given in the following table.

x	0.5	1.5	2.5	3.5	4.5	5.5
$\mathrm{P}(X = x)$	0.12	0.18	0.20	0.28	0.14	0.08

a. Determine the value of $\mathrm{E}(X^2)$. [2]

b. Find the value of $\mathrm{Var}(X)$. [3]

Markscheme

a. $\mathrm{E}(X^2) = \Sigma x^2 \bullet \mathrm{P}(X = x) = 10.37\ (= 10.4\ 3\ \text{sf})$ ***(M1)A1***

[2 marks]

b. **METHOD 1**

$\mathrm{sd}(X) = 1.44069\ldots$ ***(M1)(A1)***

$\mathrm{Var}(X) = 2.08\ (= 2.0756)$ ***A1***

METHOD 2

$\mathrm{E}(X) = 2.88\ (= 0.06 + 0.27 + 0.5 + 0.98 + 0.63 + 0.44)$ ***(A1)***

use of $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$ ***(M1)***

Note: Award ***(M1)*** only if $(\mathrm{E}(X))^2$ is used correctly.

$(\mathrm{Var}(X) = 10.37 - 8.29)$

$\mathrm{Var}(X) = 2.08\ (= 2.0756)$ ***A1***

Note: Accept 2.11.

METHOD 3

$\mathrm{E}(X) = 2.88\ (= 0.06 + 0.27 + 0.5 + 0.98 + 0.63 + 0.44)$ ***(A1)***

use of $\mathrm{Var}(X) = \mathrm{E}\left((X - \mathrm{E}(X))^2\right)$ ***(M1)***

$(0.679728 + \ldots + 0.549152)$

$\mathrm{Var}(E) = 2.08\ (= 2.0756)$ ***A1***

[3 marks]

Examiners report

a. [N/A]

b. [N/A]

The weights, in kg, of one-year-old bear cubs are modelled by a normal distribution with mean μ and standard deviation σ.

(a) Given that the upper quartile weight is 21.3 kg and the lower quartile weight is 17.1 kg, calculate the value of μ and the value of σ.

A random sample of 100 of these bear cubs is selected.

(b) Find the expected number of bear cubs weighing more than 22 kg.

Markscheme

(a) **METHOD 1**

$\mu = \frac{1}{2} \times (17.1 + 21.3)$ ***(M1)***

$\mu = 19.2$ (kg)   ***A1***

finding z value for the upper quartile $= 0.674489\text{K}$

$0.674489\text{K} = \frac{21.3-19.2}{\sigma}$ or $-0.674489\text{K} = \frac{17.1-19.2}{\sigma}$ ***M1***

$\sigma = 3.11$ (kg)   ***A1***

METHOD 2

finding z value for the upper quartile $= 0.674489\text{K}$

from symmetry the z value for a lower quartile is -0.674489K ***M1***

forming two simultaneous equations:

$-0.674489\text{K} = \frac{17.1-\mu}{\sigma}$

$0.674489\text{K} = \frac{21.3-\mu}{\sigma}$ ***M1***

solving gives:

$\mu = 19.2$ (kg)   ***A1***

$\sigma = 3.11$ (kg)   ***A1***

[4 marks]

(b) using $100 \times \text{P}(X > 22) = 100 \times 0.184241\text{K}$

$= 18$ ***A1***

Note: Accept 18.4

[1 mark]

Total [5 marks]

Examiners report

[N/A]

A company produces computer microchips, which have a life expectancy that follows a normal distribution with a mean of 90 months and a standard deviation of 3.7 months.

(a) If a microchip is guaranteed for 84 months find the probability that it will fail before the guarantee ends.

(b) The probability that a microchip does not fail before the end of the guarantee is required to be 99 %. For how many months should it be guaranteed?

(c) A rival company produces microchips where the probability that they will fail after 84 months is 0.88. Given that the life expectancy also follows a normal distribution with standard deviation 3.7 months, find the mean.

Markscheme

(a) $\text{P}(X \leqslant 84) = \text{P}(Z \leqslant -1.62...) = 0.0524$ ***(M1)A1***   ***N2***

Note: Accept 0.0526.

(b) $\text{P}(Z \leqslant z) = 0.01 \Rightarrow z = -2.326\ldots$ ***(M1)***

$\text{P}(X \leqslant x) = \text{P}(Z \leqslant z) = 0.01 \Rightarrow z = -2.326\ldots$

$x = 81.4$ (accept 81)   ***A1***   ***N2***

(c) $\mathrm{P}(X \leqslant 84) = 0.12 \Rightarrow z = -1.1749\ldots$ ***(M1)***

mean is 88.3 (accept 88) ***A1*** ***N2***

[6 marks]

Examiners report

A fair amount of students did not use their GDC directly, but used tables and more traditional methods to answer this question. Part (a) was answered correctly by most candidates using any method. A large number of candidates reversed the probabilities, i.e., failed to use a negative z value in parts (b) and (c), and hence did not obtain correct answers.

There are 75 players in a golf club who take part in a golf tournament. The scores obtained on the 18th hole are as shown in the following table.

Score	2	3	4	5	6	7
Frequency	3	15	28	17	9	3

a. One of the players is chosen at random. Find the probability that this player's score was 5 or more. [2]

b. Calculate the mean score. [2]

Markscheme

a. $\mathrm{P}(5 \text{ or more}) = \frac{29}{75}$ $(= 0.387)$ ***(M1)A1***

[2 marks]

b. mean score $= \frac{2\times3+3\times15+4\times28+5\times17+6\times9+7\times3}{75}$ ***(M1)***

$= \frac{323}{75}$ $(= 4.31)$ ***A1***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]

The annual weather-related loss of an insurance company is modelled by a random variable X with probability density function

$$f(x) = \begin{cases} \frac{2.5(200)^{2.5}}{x^{3.5}}, & x \geqslant 200 \\ 0, & \text{otherwise.} \end{cases}$$

Find the median.

Markscheme

$\int_{200}^{M} \frac{2.5(200)^{2.5}}{x^{3.5}} \mathrm{d}x = 0.5$ ***M1A1A1***

Note: Award M1 for the integral equal to 0.5

A1A1 for the correct limits.

$\frac{-200^{2.5}}{M^{2.5}} \left(\frac{-200^{2.5}}{200^{2.5}} \right) = 0.5$ ***M1A1A1***

Note: Award ***M1*** for correct integration

A1A1 for correct substitutions.

$\frac{-200^{2.5}}{M^{2.5}} + 1 = 0.5 \Rightarrow M^{2.5} = 2(200)^{2.5}$ ***(A1)***

$M = 264$ ***A1***

[8 marks]

Examiners report

Many students used incorrect limits to the integral, although many did correctly let the integral equal to 0.5.

Natasha lives in Chicago and has relatives in Nashville and St. Louis.

Each time she visits her relatives, she either flies or drives.

When travelling to Nashville, the probability that she drives is $\frac{4}{5}$, and when travelling to St. Louis, the probability that she flies is $\frac{1}{3}$.

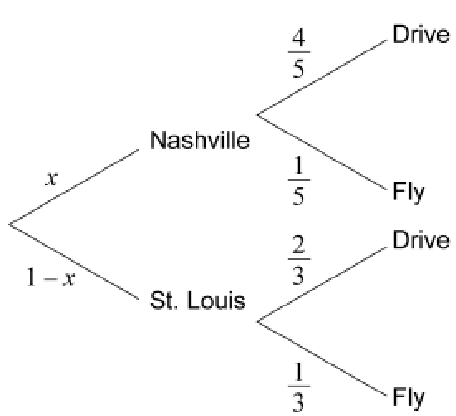
Given that the probability that she drives when visiting her relatives is $\frac{13}{18}$, find the probability that for a particular trip,

a. she travels to Nashville; [3]

b. she is on her way to Nashville, given that she is flying. [3]

Markscheme

a.



attempt to set up the problem using a tree diagram and/or an equation, with the unknown x ***M1***

$\frac{4}{5}x + \frac{2}{3}(1 - x) = \frac{13}{18}$ ***A1***

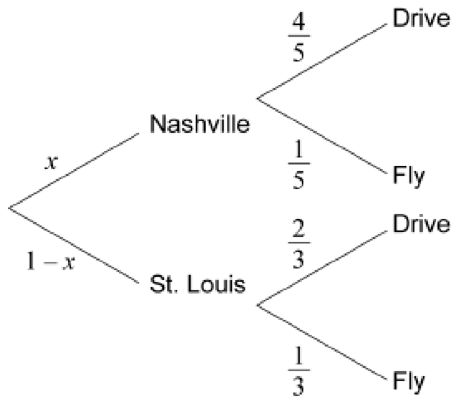
$\frac{4x}{5} - \frac{2x}{3} = \frac{13}{18} - \frac{2}{3}$

$\frac{2x}{15} = \frac{1}{18}$

$x = \frac{5}{12}$ ***A1***

[3 marks]

b.



attempt to set up the problem using conditional probability ***M1***

EITHER

$\frac{\frac{5}{12}\times\frac{1}{5}}{1-\frac{13}{18}}$ ***A1***

OR

$\frac{\frac{5}{12}\times\frac{1}{5}}{\frac{1}{12}+\frac{7}{36}}$ ***A1***

THEN

$= \frac{3}{10}$ ***A1***

[3 marks]

Total [6 marks]

Examiners report

a. [N/A]
b. [N/A]

Events A and B are such that $\mathrm{P}(A \cup B) = 0.95,\ \mathrm{P}(A \cap B) = 0.6$ and $\mathrm{P}(A|B) = 0.75$.

a. Find $\mathrm{P}(B)$. [2]

b. Find $\mathrm{P}(A)$. [2]

c. Hence show that events A' and B are independent. [2]

Markscheme

a. $\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

$\Rightarrow 0.75 = \frac{0.6}{\mathrm{P}(B)}$ ***(M1)***

$\Rightarrow \mathrm{P}(B) \left(= \frac{0.6}{0.75}\right) = 0.8$ ***A1***

[2 marks]

b. $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\Rightarrow 0.95 = \mathrm{P}(A) + 0.8 - 0.6$ ***(M1)***

$\Rightarrow \mathrm{P}(A) = 0.75$ ***A1***

[2 marks]

c. **METHOD 1**

$\mathrm{P}(A'|B) = \frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)} = \frac{0.2}{0.8} = 0.25$ ***A1***

$\mathrm{P}(A'|B) = \mathrm{P}(A')$ ***R1***

hence A' and B are independent ***AG***

Note: If there is evidence that the student has calculated $\mathrm{P}(A' \cap B) = 0.2$ by assuming independence in the first place, award ***A0R0***.

METHOD 2

EITHER

$\mathrm{P}(A) = \mathrm{P}(A|B)$ ***A1***

OR

$\mathrm{P}(A) \times \mathrm{P}(B) = 0.75 \times 0.80 = 0.6 = \mathrm{P}(A \cap B)$ ***A1***

THEN

A and B are independent ***R1***

hence A' and B are independent ***AG***

METHOD 3

$\mathrm{P}(A') \times \mathrm{P}(B) = 0.25 \times 0.80 = 0.2$ ***A1***

$\mathrm{P}(A') \times \mathrm{P}(B) = \mathrm{P}(A' \cap B)$ ***R1***

hence A' and B are independent ***AG***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The random variable X follows a Poisson distribution with mean λ.

(a) Find λ if $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.123$.

(b) With this value of λ, find $\mathrm{P}(0 < X < 9)$.

Markscheme

(a) required to solve $\mathrm{e}^{-\lambda} + \lambda\mathrm{e}^{-\lambda} = 0.123$ ***M1A1***

solving to obtain $\lambda = 3.63$ ***A2*** ***N2***

Note: Award ***A2*** if an additional negative solution is seen but ***A0*** if only a negative solution is seen.

(b) $\mathrm{P}(0 < X < 9)$

$= \mathrm{P}(X \leqslant 8) - \mathrm{P}(X = 0)$ (or equivalent) ***(M1)***

$= 0.961$ ***A1***

[6 marks]

Examiners report

Part (a) - Well done by most, although there were some answers that ignored the requirement of mathematical notation.

Part (b) - Not successfully answered by many. The main problem was not correctly interpreting the inequalities in the probability.

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{(x+1)^3}{60}, & \text{for } 1 \leqslant x \leqslant 3 \\ 0, & \text{otherwise.} \end{cases},$$

Find

(a) $\mathrm{P}(1.5 \leqslant X \leqslant 2.5)$;

(b) $\mathrm{E}(X)$;

(c) the median of X .

Markscheme

(a) $\int_{1.5}^{2.5} \frac{(x+1)^3}{60} \mathrm{d}x = 0.4625 \quad (= 0.463)$ ***M1A1***

(b) $\mathrm{E}(X) = \int_1^3 \frac{x(x+1)^3}{60} \mathrm{d}x = 2.31$ ***M1A1***

(c) $\int_1^m \frac{(x+1)^3}{60} \mathrm{d}x = 0.5$ ***M1***

$\left[\frac{(x+1)^4}{240}\right]_1^m = 0.5$ ***(A1)***

$m = 2.41$ ***A1***

[7 marks]

Examiners report

Parts (a) and (b) were reasonably well done in general but (c) caused problems for many candidates where several misconceptions regarding the median were seen. The expectation was that candidates would use their GDCs to solve (a) and (b), and possibly even (c), although in the event most candidates did the integrations by hand. Those candidates using their GDCs made fewer mistakes in general than those doing the integrations analytically.

The random variable X has the distribution $\text{Po}(m)$.
Given that $\text{P}(X = 5) = \text{P}(X = 3) + \text{P}(X = 4)$, find

a. the value of m ; [3]

b. P (X> 2) . [2]

Markscheme

a. $\text{P}(X = 5) = \text{P}(X = 3) + \text{P}(X = 4)$
$\frac{\text{e}^{-m}m^5}{5!} = \frac{\text{e}^{-m}m^3}{3!} + \frac{\text{e}^{-m}m^4}{4!}$ ***M1(A1)***
$m^2 - 5m - 20 = 0$
$\Rightarrow m = \frac{5+\sqrt{105}}{2} = (7.62)$ ***A1***
[3 marks]

b. $\text{P}(X > 2) = 1 - \text{P}(X \leqslant 2)$ ***(M1)***
$= 1 - 0.018...$
$= 0.982$ ***A1***
[2 marks]

Examiners report

a. Again this proved to be a successful question for many candidates with a good proportion of wholly correct answers seen. It was good to see students making good use of the calculator.

b. Again this proved to be a successful question for many candidates with a good proportion of wholly correct answers seen. It was good to see students making good use of the calculator.

The fish in a lake have weights that are normally distributed with a mean of 1.3 kg and a standard deviation of 0.2 kg.

a. Determine the probability that a fish which is caught weighs less than 1.4 kg. [1]

b. John catches 6 fish. Calculate the probability that at least 4 of the fish weigh more than 1.4 kg. [3]

c. Determine the probability that a fish which is caught weighs less than 1 kg, given that it weighs less than 1.4 kg. [2]

Markscheme

a. $\mathrm{P}(x < 1.4) = 0.691 \quad \text{(accept 0.692)}$ ***A1***

[1 mark]

b. **METHOD 1**

$y \sim \mathrm{B}(6,\, 0.3085\ldots)$ ***(M1)***

$\mathrm{P}(Y \geqslant 4) = 1 - \mathrm{P}(Y \leqslant 3)$ ***(M1)***

$= 0.0775 \quad \text{(accept 0.0778 if 3sf approximation from (a) used)}$ ***A1***

METHOD 2

$X \sim \mathrm{B}(6,\, 0.6914\ldots)$ ***(M1)***

$\mathrm{P}(X \leqslant 2)$ ***(M1)***

$= 0.0775 \quad \text{(accept 0.0778 if 3sf approximation from (a) used)}$ ***A1***

[3 marks]

c. $\mathrm{P}(x < 1 | x < 1.4) = \frac{\mathrm{P}(x<1)}{\mathrm{P}(x<1.4)}$ ***M1***

$= \frac{0.06680\ldots}{0.6914\ldots}$

$= 0.0966 \quad \text{(accept 0.0967)}$ ***A1***

[2 marks]

Examiners report

a. Part (a) was almost universally correctly answered, albeit with an accuracy penalty in some cases. In (b) it was generally recognised that the distribution was binomial, but with some wavering about the correct value of the parameter p. Part (c) was sometimes answered correctly, but not with much confidence.

b. Part (a) was almost universally correctly answered, albeit with an accuracy penalty in some cases. In (b) it was generally recognised that the distribution was binomial, but with some wavering about the correct value of the parameter p. Part (c) was sometimes answered correctly, but not with much confidence.

c. Part (a) was almost universally correctly answered, albeit with an accuracy penalty in some cases. In (b) it was generally recognised that the distribution was binomial, but with some wavering about the correct value of the parameter p. Part (c) was sometimes answered correctly, but not with much confidence.

A company produces rectangular sheets of glass of area 5 square metres. During manufacturing these glass sheets flaws occur at the rate of 0.5 per 5 square metres. It is assumed that the number of flaws per glass sheet follows a Poisson distribution.

Glass sheets with no flaws earn a profit of \$5. Glass sheets with at least one flaw incur a loss of \$3.

This company also produces larger glass sheets of area 20 square metres. The rate of occurrence of flaws remains at 0.5 per 5 square metres.

A larger glass sheet is chosen at random.

a. Find the probability that a randomly chosen glass sheet contains at least one flaw. [3]

b. Find the expected profit, P dollars, per glass sheet. [3]

c. Find the probability that it contains no flaws. [2]

Markscheme

a. $\mathrm{X} \sim \mathrm{Po}(0.5)$ ***(A1)***

$\mathrm{P}(X \geqslant 1) = 0.393 \; (= 1 - \mathrm{e}^{-0.5})$ ***(M1)A1***

[3 marks]

b. $\mathrm{P}(X = 0) = 0.607\ldots$ ***(A1)***

$\mathrm{E}(P) = (0.607\ldots \times 5) - (0.393\ldots \times 3)$ ***(M1)***

the expected profit is $1.85 per glass sheet ***A1***

[3 marks]

c. $Y \sim \mathrm{Po}(2)$ ***(M1)***

$\mathrm{P}(Y = 0) = 0.135 \; (= \mathrm{e}^{-2})$ ***A1***

[2 marks]

Examiners report

a. Part (a) was reasonably well done. Some candidates calculated $\mathrm{P}(X = 1)$.

b. Part (b) was not as well done as expected with a surprising number of candidates calculating $5\mathrm{P}(X = 0) + 3\mathrm{P}(X \geqslant 1)$ rather than $5\mathrm{P}(X = 0) - 3\mathrm{P}(X \geqslant 1)$.

c. Part (c) was very well done.

Each of the 25 students in a class are asked how many pets they own. Two students own three pets and no students own more than three pets. The mean and standard deviation of the number of pets owned by students in the class are $\frac{18}{25}$ and $\frac{24}{25}$ respectively.

Find the number of students in the class who do not own a pet.

Markscheme

METHOD 1

let *p* have no pets, *q* have one pet and *r* have two pets ***(M1)***

$p + q + r + 2 = 25$ ***(A1)***

$0p + 1q + 2r + 6 = 18$ ***A1***

Note: Accept a statement that there are a total of 12 pets.

attempt to use variance equation, or evidence of trial and error ***(M1)***

$\frac{0p+1q+4r+18}{25} - \left(\frac{18}{25}\right)^2 = \left(\frac{24}{25}\right)^2$ ***(A1)***

attempt to solve a system of linear equations ***(M1)***

$p = 14$ ***A1***

METHOD 2

x	0	1	2	3
$\mathrm{P}(X = x)$	p	q	r	$\frac{2}{25}$

(M1)

$p + q + r + \frac{2}{25} = 1$ ***(A1)***

$q + 2r + \frac{6}{25} = \frac{18}{25} \left(\Rightarrow q + 2r = \frac{12}{25}\right)$ ***A1***

$q + 4r + \frac{18}{25} - \left(\frac{18}{25}\right)^2 = \frac{576}{625} \left(\Rightarrow q + 4r = \frac{18}{25}\right)$ ***(M1)(A1)***

$q = \frac{6}{25},\ r = \frac{3}{25}$ ***(M1)***

$p = \frac{14}{25}$ ***A1***

so 14 have no pets

[7 marks]

Examiners report

[N/A]

The heights of students in a single year group in a large school can be modelled by a normal distribution.

It is given that 40% of the students are shorter than 1.62 m and 25% are taller than 1.79 m.

Find the mean and standard deviation of the heights of the students.

Markscheme

let the heights of the students be X

$\mathrm{P}(X < 1.62) = 0.4,\ \mathrm{P}(X > 1.79) = 0.25$ ***M1***

Note: Award ***M1*** for either of the probabilities above.

$\mathrm{P}\left(Z < \frac{1.62-\mu}{\sigma}\right) = 0.4,\ \mathrm{P}\left(Z < \frac{1.79-\mu}{\sigma}\right) = 0.75$ ***M1***

Note: Award ***M1*** for either of the expressions above.

$\frac{1.62-\mu}{\sigma} = -0.2533\ldots,\ \frac{1.79-\mu}{\sigma} = 0.6744\ldots$ ***M1A1***

Note: ***A1*** for both values correct.

$\mu = 1.67\ (\mathrm{m}),\ \sigma = 0.183\ (\mathrm{m})$ ***A1A1***

Note: Accept answers that round to 1.7 (m) and 0.18 (m).

Note: Accept answers in centimetres.

[6 marks]

Examiners report

A large number of good solutions in this question, although candidates failing on the question failed at different stages. A number did not standardise the distribution correctly, and there were others who were unable to correctly solve the simultaneous equations. There were a notable number of otherwise good candidates who were unable to attempt the question, even though it is of a very standard type.

The duration of direct flights from London to Singapore in a particular year followed a normal distribution with mean μ and standard deviation σ.

92% of flights took under 13 hours, while only 12% of flights took under 12 hours 35 minutes.
Find μ and σ to the nearest minute.

Markscheme

$\mathrm{P}\left(Z < \frac{780-\mu}{\sigma}\right) = 0.92$ and $\mathrm{P}\left(Z < \frac{755-\mu}{\sigma}\right) = 0.12$ ***(M1)***

use of inverse normal ***(M1)***

$\Rightarrow \frac{780-\mu}{\sigma} = 1.405\ldots$ and $\frac{755-\mu}{\sigma} = -1.174\ldots$ ***(A1)***

solving simultaneously ***(M1)***

Note: Award ***M1*** for attempting to solve an incorrect pair of equations *eg,* inverse normal not used.

$\mu = 766.385$
$\sigma = 9.6897$
$\mu = 12$ hrs 46 mins $(= 766$ mins$)$ ***A1***
$\sigma = 10$ mins ***A1***
[6 marks]

Examiners report

Generally well done. Most candidates made correct use of the symmetry of the normal curve and the inverse normal to set up a correct pair of equations involving μ and σ. A few candidates expressed equations containing the GDC command term invNorm.

A few candidates did not express their answers correct to the nearest minute and a few candidates performed erroneous conversions from hours to minutes.

Emily walks to school every day. The length of time this takes can be modelled by a normal distribution with a mean of 11 minutes and a standard deviation of 3 minutes. She is late if her journey takes more than 15 minutes.

a. Find the probability she is late next Monday. [2]

b. Find the probability she is late at least once during the next week (Monday to Friday). [3]

Markscheme

a. Let X represent the length of time a journey takes on a particular day.

$P(X > 15) = 0.0912112819\ldots = 0.0912$ ***(M1)A1***

b. Use of correct Binomial distribution ***(M1)***

$N \sim B(5, 0.091\ldots)$

$1 - 0.0912112819\ldots = 0.9087887181\ldots$

$1 - (0.9087887181\ldots)^5 = 0.380109935\ldots = 0.380$ ***(M1)A1***

Note: Allow answers to be given as percentages.

[5 marks]

Examiners report

a. There were many good answers to this question. Some students lost accuracy marks by early rounding. Some students struggled with the Binomial distribution.

b. There were many good answers to this question. Some students lost accuracy marks by early rounding. Some students struggled with the Binomial distribution.

Over a one month period, Ava and Sven play a total of n games of tennis.

The probability that Ava wins any game is 0.4. The result of each game played is independent of any other game played.

Let X denote the number of games won by Ava over a one month period.

(a) Find an expression for $P(X = 2)$ in terms of n.

(b) If the probability that Ava wins two games is 0.121 correct to three decimal places, find the value of n.

Markscheme

(a) $X \sim \mathrm{B}(n,\ 0.4)$ ***(A1)***

Using $\mathrm{P}(X = x) = \binom{n}{r}(0.4)^x(0.6)^{n-x}$ ***(M1)***

$\mathrm{P}(X = 2) = \binom{n}{2}(0.4)^2(0.6)^{n-2} \quad \left(= \frac{n(n-1)}{2}(0.4)^2(0.6)^{n-2}\right)$ ***A1 N3***

(b) $\mathrm{P}(X = 2) = 0.121$ ***A1***

Using an appropriate method (including trial and error) to solve their equation. ***(M1)***

$n = 10$ ***A1 N2***

Note: Do not award the last ***A1*** if any other solution is given in their final answer.

[6 marks]

Examiners report

Part (a) was generally well done. The most common error was to omit the binomial coefficient *i.e.* not identifying that the situation is described by a binomial distribution.

Finding the correct value of n in part (b) proved to be more elusive. A significant proportion of candidates attempted algebraic approaches and seemingly did not realise that the equation could only be solved numerically. Candidates who obtained $n = 10$ often accomplished this by firstly attempting to solve the equation algebraically before 'resorting' to a GDC approach. Some candidates did not specify their final answer as an integer while others stated $n = 1.76$ as their final answer.

The number of birds seen on a power line on any day can be modelled by a Poisson distribution with mean 5.84.

a. Find the probability that during a certain seven-day week, more than 40 birds have been seen on the power line. [2]

b. On Monday there were more than 10 birds seen on the power line. Show that the probability of there being more than 40 birds seen on the power line from that Monday to the following Sunday, inclusive, can be expressed as: [5]

$\frac{\mathrm{P}(X>40)+\sum_{r=11}^{40}\mathrm{P}(X=r)\mathrm{P}(Y>40-r)}{\mathrm{P}(X>10)}$ where $X \sim \mathrm{Po}(5.84)$ and $Y \sim \mathrm{Po}(35.04)$.

Markscheme

a. mean for week is 40.88 ***(A1)***

$\mathrm{P}(S > 40) = 1 - \mathrm{P}(S \leqslant 40) = 0.513$ ***A1***

[2 marks]

b. $\frac{\text{probability there were more than 10 on Monday AND more than 40 over the week}}{\text{probability there were more than 10 on Monday}}$ ***M1***

possibilities for the numerator are:

there were more than 40 birds on the power line on Monday ***R1***

11 on Monday and more than 29 over the course of the next 6 days ***R1***

12 on Monday and more than 28 over the course of the next 6 days … until

40 on Monday and more than 0 over the course of the next 6 days ***R1***

hence if X is the number on the power line on Monday and Y, the number on the power line Tuesday – Sunday then the numerator is ***M1***

$\mathrm{P}(X > 40) + \mathrm{P}(X = 11) \times \mathrm{P}(Y > 29) + \mathrm{P}(X = 12) \times \mathrm{P}(Y > 28) + \ldots$

$+\mathrm{P}(X = 40) \times \mathrm{P}(Y > 0)$

$= \mathrm{P}(X > 40) + \sum_{r=11}^{40} \mathrm{P}(X = r)\mathrm{P}(Y > 40 - r)$

hence solution is $\frac{\mathrm{P}(X>40)+\sum_{r=11}^{40}\mathrm{P}(X=r)\mathrm{P}(Y>40-r)}{\mathrm{P}(X>10)}$ ***AG***

[5 marks]

Examiners report

[N/A]

a. [N/A]

Mr Lee is planning to go fishing this weekend. Assuming that the number of fish caught per hour follows a Poisson distribution with mean 0.6, find

(a) the probability that he catches at least one fish in the first hour;

(b) the probability that he catches exactly three fish if he fishes for four hours;

(c) the number of complete hours that Mr Lee needs to fish so that the probability of catching more than two fish exceeds 80 %.

Markscheme

(a) X ~ $\mathrm{Po}(0.6)$

$\mathrm{P}(X \geqslant 1) = 1 - \mathrm{P}(X = 0)$ ***M1***

$= 0.451$ ***A1*** ***N1***

(b) Y \~ $\mathrm{Po}(2.4)$ ***(M1)***

$\mathrm{P}(Y = 3) = 0.209$ ***A1***

(c) Z \~ $\mathrm{Po}(0.6n)$ ***(M1)***

$\mathrm{P}(Z \geqslant 3) = 1 - \mathrm{P}(Z \leqslant 2) > 0.8$ ***(M1)***

Note: Only one of these ***M1*** marks may be implied.

$n \geqslant$ 7.132... (hours)

so, Mr Lee needs to fish for at least 8 complete hours ***A1*** ***N2***

Note: Accept a shown trial and error method that leads to a correct solution.

[7 marks]

Examiners report

It was clear that many students had not been taught the topic and were consequently unable to make an attempt at the question. Of those students who were able to start, common errors were in a misunderstanding of the language. Many had difficulties in part (c) and “at least” in part (a) was sometimes misinterpreted.

Part A. (a) A box of biscuits is considered to be underweight if it weighs less than 228 grams. It is known that the weights of these boxes of biscuits are normally distributed with a mean of 231 grams and a standard deviation of 1.5 grams. [11]

What is the probability that a box is underweight?

(b) The manufacturer decides that the probability of a box being underweight should be reduced to 0.002.
(i) Bill's suggestion is to increase the mean and leave the standard deviation unchanged. Find the value of the new mean.
(ii) Sarah's suggestion is to reduce the standard deviation and leave the mean unchanged. Find the value of the new standard deviation.
(c) After the probability of a box being underweight has been reduced to 0.002, a group of customers buys 100 boxes of biscuits. Find the probability that at least two of the boxes are underweight.

Part B. There are six boys and five girls in a school tennis club. A team of two boys and two girls will be selected to represent the school in a tennis competition. [10]

(a) In how many different ways can the team be selected?
(b) Tim is the youngest boy in the club and Anna is the youngest girl. In how many different ways can the team be selected if it must include both of them?
(c) What is the probability that the team includes both Tim and Anna?
(d) Fred is the oldest boy in the club. Given that Fred is selected for the team, what is the probability that the team includes Tim or Anna, but not both?

Markscheme

Part A. (a) $X \sim \mathrm{N}(231, 1.5^2)$

$\mathrm{P}(X < 228) = 0.0228$ ***(M1)A1***

Note: Accept 0.0227.

[2 marks]

(b) (i) $X \sim \mathrm{N}(\mu, 1.5^2)$

$\mathrm{P}(X < 228) = 0.002$

$\frac{228-\mu}{1.5} = -2.878\ldots$ ***M1A1***

$\mu = 232$ grams ***A1 N3***

(ii) $X \sim \mathrm{N}(231, \sigma^2)$

$\frac{228-231}{\sigma} = -2.878\ldots$ ***M1A1***

$\sigma = 1.04$ grams ***A1 N3***

[6 marks]

(c) $X \sim \mathrm{B}(100, 0.002)$ ***(M1)***

$\mathrm{P}(X \leqslant 1) = 0.982\ldots$ ***(A1)***

$\mathrm{P}(X \geqslant 2) = 1 - \mathrm{P}(X \leqslant 1) = 0.0174$ ***A1***

[3 marks]

Total [11 marks]

Part B. (a) Boys can be chosen in $\frac{6\times5}{2} = 15$ ways ***(A1)***

Girls can be chosen in $\frac{5\times4}{2} = 10$ ways ***(A1)***

Total $= 15 \times 10 = 150$ ways ***A1***

[3 marks]

(b) Number of ways $= 5 \times 4 = 20$ ***(M1)A1***

[2 marks]

(c) $\frac{20}{150} \left(= \frac{2}{15}\right)$ ***A1***

[1 mark]

(d) **METHOD 1**

$\mathrm{P}(T) = \frac{1}{5}$; $\mathrm{P}(A) = \frac{2}{5}$ ***A1***

$\mathrm{P}(T \text{ or } A \text{ but not both}) = \mathrm{P}(T) \times \mathrm{P}(A') + \mathrm{P}(T') \times \mathrm{P}(A)$ ***M1A1***

$= \frac{1}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{2}{5} = \frac{11}{25}$ ***A1***

METHOD 2

Number of selections including Fred $= 5 \times \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 50$ ***A1***

Number of selections including Tim but not Anna $= \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 6$ ***A1***

Number of selections including Anna but not Tim $= 4 \times 4 = 16$

Note: Both statements are needed to award ***A1***.

$\mathrm{P}(T \text{ or } A \text{ but not both}) = \frac{6+16}{50} = \frac{11}{25}$ ***M1A1***

[4 marks]

Total [10 marks]

Examiners report

Part A: Part A was well done by many candidates although an arithmetic penalty was often awarded in (b)(i) for giving the new value of the mean to too many significant figures.

Part B: Candidates are known, however, to be generally uncomfortable with combinatorial mathematics and Part B caused problems for many candidates. Even some of those candidates who solved (a) and (b) correctly were then unable to deduce the answer to (c), sometimes going off on some long-winded solution which invariably gave the wrong answer. Very few correct solutions were seen to (d).

Consider two events A and B such that $\mathrm{P}(A) = k,\ \mathrm{P}(B) = 3k,\ \mathrm{P}(A \cap B) = k^2$ and $\mathrm{P}(A \cup B) = 0.5$.

a. Calculate k; [3]

b. Find $\mathrm{P}(A' \cap B)$. [3]

Markscheme

a. use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***M1***

$0.5 = k + 3k - k^2$ ***A1***

$k^2 - 4k + 0.5 = 0$

$k = 0.129$ ***A1***

Note: Do not award the final ***A1*** if two solutions are given.

[3 marks]

b. use of $\mathrm{P}(A' \cap B) = \mathrm{P}(B) - \mathrm{P}(A \cap B)$ or alternative ***(M1)***

$\mathrm{P}(A' \cap B) = 3k - k^2$ ***(A1)***

$= 0.371$ ***A1***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

When carpet is manufactured, small faults occur at random. The number of faults in Premium carpets can be modelled by a Poisson distribution with mean 0.5 faults per 20 m^2. Mr Jones chooses Premium carpets to replace the carpets in his office building. The office building has 10 rooms, each with the area of 80 m^2.

a. Find the probability that the carpet laid in the first room has fewer than three faults. [3]

b. Find the probability that exactly seven rooms will have fewer than three faults in the carpet. [3]

Markscheme

a. $\lambda = 4 \times 0.5$ ***(M1)***

$\lambda = 2$ ***(A1)***

$\mathrm{P}(X \leqslant 2) = 0.677$ ***A1***

[3 marks]

b. $Y \sim B(10,\ 0,677)$ ***(M1)(A1)***

$\mathrm{P}(Y = 7) = 0.263$ ***A1***

Note: Award ***M1*** for clear recognition of binomial distribution.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

The probability density function of the continuous random variable X is given by

$$f(x) = \begin{cases} k2^{\frac{1}{x}}, & 1 \leqslant x \leqslant 2 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant. Find the expected value of X .

Markscheme

$k\int_1^2 2^{\frac{1}{x}}\mathrm{d}x = 1 \Rightarrow k = \frac{1}{\int_1^2 2^{\frac{1}{x}}\mathrm{d}x}$ $(= 0.61556...)$ ***(M1)(A1)***

$\mathrm{E}(X) = k\int_1^2 x2^{\frac{1}{x}}\mathrm{d}x = 2.39....k$ or 1.47 ***M1A1***

Note: Condone missing dx in any part of the question.

[4 marks]

Examiners report

This question was well attempted by most candidates. However many were not alert for the necessity of using GDC to calculate the definite integrals and wasted time trying to obtain these values using standard calculus methods without success.

The mean number of squirrels in a certain area is known to be 3.2 squirrels per hectare of woodland. Within this area, there is a 56 hectare woodland nature reserve. It is known that there are currently at least 168 squirrels in this reserve.

Assuming the population of squirrels follow a Poisson distribution, calculate the probability that there are more than 190 squirrels in the reserve.

Markscheme

X is number of squirrels in reserve

X ~ Po(179.2) ***A1***

Note: Award ***A1*** if 179.2 or 56 × 3.2 seen or implicit in future calculations.

recognising conditional probability ***M1***

P(X > 190 | X ≥ 168)

$= \frac{\mathrm{P}(X>190)}{\mathrm{P}(X\geqslant168)} = \left(\frac{0.19827\ldots}{0.80817\ldots}\right)$ ***(A1)(A1)***

= 0.245 ***A1***

[5 marks]

Examiners report

[N/A]

A set of 15 observations has mean 11.5 and variance 9.3. One observation of 22.1 is considered unreliable and is removed. Find the mean and variance of the remaining 14 observations.

Markscheme

$\frac{\sum_{i=1}^{15} x_i}{15} = 11.5 \Rightarrow \sum_{i=1}^{15} x_i = 172.5$ ***(A1)***

new mean $= \frac{172.5-22.1}{14}$ ***(M1)***

= 10.7428… = 10.7 (3sf) ***A1***

$\frac{\sum_{i=1}^{15} x_i^2}{15} - 11.5^2 = 9.3$ ***(M1)***

$\Rightarrow \sum_{i=1}^{15} x_i^2 = 2123.25$

new variance $= \frac{2123.25-22.1^2}{14} - (10.7428...)^2$ ***(M1)***

= 1.37 (3sf) ***A1***

[6 marks]

Examiners report

Most candidates were successful in finding the correct value of the mean; however, the variance caused many difficulties. Many candidates affirmed that there were no differences in the variance as it remained constant; some others got wrong results due to premature rounding of figures. Many candidates lost the final mark because they rounded their answers prematurely, resulting in a very inaccurate answer to this question.

The probability density function of a random variable *X* is defined as:

$$f(x) = \begin{cases} ax\cos x, & 0 \le x \le \frac{\pi}{2}, \text{where } a \in \mathbb{R} \\ 0, & \text{elsewhere} \end{cases}$$

(a) Show that $a = \frac{2}{\pi-2}$.

(b) Find $\mathrm{P}\left(X < \frac{\pi}{4}\right)$.

(c) Find:

(i) the mode of *X*;

(ii) the median of *X*.

(d) Find $\mathrm{P}\left(X < \frac{\pi}{8} | X < \frac{\pi}{4}\right)$.

Markscheme

(a) $a\int_0^{\frac{\pi}{2}} x\cos x \mathrm{d}x = 1$ ***(M1)***

integrating by parts:

$u = x \quad v' = \cos x$ ***M1***

$u' = 1 \quad v = \sin x$

$\int x\cos x \mathrm{d}x = x\sin x + \cos x$ ***A1***

$[x\sin x + \cos x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$ ***A1***

$a = \frac{1}{\frac{\pi}{2}-1}$ ***A1***

$= \frac{2}{\pi-2}$ ***AG***

[5 marks]

(b) $\mathrm{P}\left(X < \frac{\pi}{4}\right) = \frac{2}{\pi-2}\int_0^{\frac{\pi}{4}} x\cos x\,\mathrm{d}x = 0.460$ ***(M1)A1***

Note: Accept $\frac{2}{\pi-2}\left(= \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1\right)$ or equivalent

[2 marks]

(c) (i) $\text{mode} = 0.860$ ***A1***

(x-value of a maximum on the graph over the given domain)

(ii) $\frac{2}{\pi-2}\int_0^m x\cos x\,\mathrm{d}x = 0.5$ ***(M1)***

$\int_0^m x\cos x\,\mathrm{d}x = \frac{\pi-2}{4}$

$m\sin m + \cos m - 1 = \frac{\pi-2}{4}$ ***(M1)***

$\text{median} = 0.826$ ***A1***

Note: Do not accept answers containing additional solutions.

[4 marks]

(d) $\mathrm{P}\left(X < \frac{\pi}{8} | X < \frac{\pi}{4}\right) = \frac{\mathrm{P}\left(X<\frac{\pi}{8}\right)}{\mathrm{P}\left(X<\frac{\pi}{4}\right)}$ ***M1***

$= \frac{0.129912}{0.459826}$

$= 0.283$ ***A1***

[2 marks]

Total [13 marks]

Examiners report

[N/A]

The random variable *X* has a binomial distribution with parameters *n* and *p*.

It is given that E(*X*) = 3.5.

a. Find the least possible value of *n*. [2]

b. It is further given that P(*X* ≤ 1) = 0.09478 correct to 4 significant figures. [5]

Determine the value of *n* and the value of *p*.

Markscheme

a. *np* = 3.5 ***(A1)***

p ≤ 1 ⇒ least *n* = 4 ***A1***

[2 marks]

b. $(1 - p)^n + np(1 - p)^{n-1} = 0.09478$ ***M1A1***

attempt to solve above equation with $np = 3.5$ ***(M1)***

$n = 12$, $p = \frac{7}{24}$ $(=0.292)$ ***A1A1***

Note: Do not accept n as a decimal.

[5 marks]

Examiners report

a. [N/A]
b. [N/A]

A discrete random variable X follows a Poisson distribution $\mathrm{Po}(\mu)$.

a. Show that $\mathrm{P}(X = x + 1) = \frac{\mu}{x+1} \times \mathrm{P}(X = x),\ x \in \mathbb{N}$. [3]

b. Given that $\mathrm{P}(X = 2) = 0.241667$ and $\mathrm{P}(X = 3) = 0.112777$, use part (a) to find the value of μ. [3]

Markscheme

a. **METHOD 1**

$\mathrm{P}(X = x + 1) = \frac{\mu^{x+1}}{(x+1)!}\mathrm{e}^{-\mu}$ ***A1***

$= \frac{\mu}{x+1} \times \frac{\mu^x}{x!}\mathrm{e}^{-\mu}$ ***M1A1***

$= \frac{\mu}{x+1} \times \mathrm{P}(X = x)$ ***AG***

METHOD 2

$\frac{\mu}{x+1} \times \mathrm{P}(X = x) = \frac{\mu}{x+1} \times \frac{\mu^x}{x!}\mathrm{e}^{-\mu}$ ***A1***

$= \frac{\mu^{x+1}}{(x+1)!}\mathrm{e}^{-\mu}$ ***M1A1***

$= \mathrm{P}(X = x + 1)$ ***AG***

METHOD 3

$\frac{\mathrm{P}(X=x+1)}{\mathrm{P}(X=x)} = \frac{\frac{\mu^{x+1}}{(x+1)!}\mathrm{e}^{-\mu}}{\frac{\mu^x}{x!}\mathrm{e}^{-\mu}}$ ***(M1)***

$= \frac{\mu^{x+1}}{\mu^x} \times \frac{x!}{(x+1)!}$ ***A1***

$= \frac{\mu}{x+1}$ ***A1***

and so $\mathrm{P}(X = x + 1) = \frac{\mu}{x+1} \times \mathrm{P}(X = x)$ ***AG***

[3 marks]

b. $\mathrm{P}(X = 3) = \frac{\mu}{3} \bullet \mathrm{P}(X = 2)$ $\left(0.112777 = \frac{\mu}{3} \bullet 0.241667\right)$ ***A1***

attempting to solve for μ ***(M1)***

$\mu = 1.40$ ***A1***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

A random variable X is normally distributed with mean 3 and variance 2^2.

a. Find $\mathrm{P}(0 \leqslant X \leqslant 2)$. [2]

b. Find $\mathrm{P}(|X| > 1)$. [3]

c. If $\mathrm{P}(X > c) = 0.44$, find the value of c. [2]

Markscheme

a. $\mathrm{P}(0 \leqslant X \leqslant 2) = 0.242$ ***(M1)A1***

[2 marks]

b. **METHOD 1**

$\mathrm{P}(|X| > 1) = \mathrm{P}(X < -1) + \mathrm{P}(X > 1)$ ***(M1)***

$= 0.02275\ldots + 0.84134\ldots$ ***(A1)***

$= 0.864$ ***A1***

METHOD 2

$\mathrm{P}(|X| > 1) = 1 - \mathrm{P}(-1 < X < 1)$ ***(M1)***

$= 1 - 0.13590\ldots$ ***(A1)***

$= 0.864$ ***A1***

[3 marks]

c. $c = 3.30$ ***(M1)A1***

[2 marks]

Examiners report

a. Part (a) was generally well done. In each question part, a number of candidates could have benefited from producing a labelled sketch of the situation.

b. Part (b) was not well done with many candidates not knowing what $\mathrm{P}(|X| > 1)$ represents. In each question part, a number of candidates could have benefited from producing a labelled sketch of the situation.

c. In part (c), a number of candidates did not recognise that $\mathrm{P}(X > c) = 1 - \mathrm{P}(X \leqslant c)$. In each question part, a number of candidates could have benefited from producing a labelled sketch of the situation.

The random variable *X* follows a Poisson distribution with mean *m* and satisfies

$$\mathrm{P}(X = 1) + \mathrm{P}(X = 3) = \mathrm{P}(X = 0) + \mathrm{P}(X = 2).$$

(a) Find the value of *m* correct to four decimal places.

(b) For this value of *m*, calculate $\mathrm{P}(1 \leqslant X \leqslant 2)$.

Markscheme

(a) $\mathrm{P}(X = 1) + \mathrm{P}(X = 3) = \mathrm{P}(X = 0) + \mathrm{P}(X = 2)$

$m\mathrm{e}^{-m} + \frac{m^3\mathrm{e}^{-m}}{6} = \mathrm{e}^{-m} + \frac{m^2\mathrm{e}^{-m}}{2}$ ***(M1)(A1)***

$m^3 - 3m^2 + 6m - 6 = 0$ ***(M1)***

$m = 1.5961$ (4 decimal places) ***A1***

(b) $m = 1.5961 \Rightarrow \mathrm{P}(1 \leqslant X \leqslant 2) = m\mathrm{e}^{-m} + \frac{m^2\mathrm{e}^{-m}}{2} = 0.582$ ***(M1)A1***

[6 marks]

Examiners report

Most candidates correctly stated the required equation for *m*. However, many algebraic errors in the simplification of this equation led to incorrect answers. Also, many candidates failed to find the value of *m* to the required accuracy, with many candidates giving answers correct to 4 sf instead of 4 dp. In part (b) many candidates did not realize that they needed to calculate $\mathrm{P}(X = 1) + \mathrm{P}(X = 2)$ and many attempts to calculate other combinations of probabilities were seen.

A student sits a national test and is told that the marks follow a normal distribution with mean 100. The student receives a mark of 124 and is told that he is at the 68^{th} percentile.
Calculate the variance of the distribution.

Markscheme

$X : \mathrm{N}(100,\ \sigma^2)$

$\mathrm{P}(X < 124) = 0.68$ ***(M1)(A1)***
$\frac{24}{\sigma} = 0.4676\ldots$ ***(M1)***

$\sigma = 51.315\ldots$ ***(A1)***

variance = 2630 ***A1***

Notes: Accept use of $\mathrm{P}(X < 124.5) = 0.68$ leading to variance = 2744.

[5 marks]

Examiners report

[N/A]

It is believed that the lifespans of Manx cats are normally distributed with a mean of 13.5 years and a variance of 9.5 years^2.

a. Calculate the range of lifespans of Manx cats whose lifespans are within one standard deviation of the mean. [2]

b. Estimate the number of Manx cats in a population of 10 000 that will have a lifespan of less than 10 years. Give your answer to the nearest whole number. [3]

Markscheme

a. $X \sim N(13.5,\ 9.5)$

$13.5 - \sqrt{9.5} < X < 13.5 + \sqrt{9.5}$ ***(M1)***

$10.4 < X < 16.6$ ***A1***

Note: Accept 6.16.

[2 marks]

b. $\mathrm{P}(X < 10) = 0.12807\ldots$ ***(M1)(A1)***

estimate is 1281 (correct to the nearest whole number). ***A1***

Note: Accept 1280.

[3 marks]

Examiners report

a. A large proportion of candidates experienced difficulties with this question. In parts (a) and (b), the most common error was to use $\sigma = 9.5$. In part (a), a large number of candidates used their range of values to then unnecessarily find the corresponding probability of that time interval occurring. In part (b), a large number of candidates used an unrealistic lower bound (a large negative value) for time.

b. A large proportion of candidates experienced difficulties with this question. In parts (a) and (b), the most common error was to use $\sigma = 9.5$. In part (a), a large number of candidates used their range of values to then unnecessarily find the corresponding probability of that time interval occurring. In part (b), a large number of candidates used an unrealistic lower bound (a large negative value) for time.

Bob measured the heights of 63 students. After analysis, he conjectured that the height, H, of the students could be modelled by a normal distribution with mean 166.5 cm and standard deviation 5 cm.

(a) Based on this assumption, estimate the number of these students whose height is at least 170 cm.

Later Bob noticed that the tape he had used to measure the heights was faulty as it started at the 5 cm mark and not at the zero mark.

(b) What are the correct values of the mean and variance of the distribution of the heights of these students?

Markscheme

(a) H \~ N(166.5, 5^2)

$\mathrm{P}(H \geqslant 170) = 0.242...$ ***(M1)(A1)***

$0.242... \times 63 = 15.2$ ***A1***

so, approximately 15 students

(b) correct mean: 161.5 (cm) ***A1***

variance remains the same, *i.e.* 25 (cm^2) ***A2***

[6 marks]

Examiners report

A surprising number of students lacked the basic knowledge of the normal distribution and were unable to answer the first part of this question. Those students who showed a knowledge of the topic tended to answer the question well. In part (b) many students either had a misunderstanding of the difference between variance and standard deviation, or did not read the question properly.

At the start of each week, Eric and Marina pick a night at random on which they will watch a movie.

If they choose a Saturday night, the probability that they watch a French movie is $\frac{7}{9}$ and if they choose any other night the probability that they watch a French movie is $\frac{4}{9}$.

a. Find the probability that they watch a French movie. [3]

b. Given that last week they watched a French movie, find the probability that it was on a Saturday night. [2]

Markscheme

a. $\mathrm{P}(F) = \left(\frac{1}{7} \times \frac{7}{9}\right) + \left(\frac{6}{7} \times \frac{4}{9}\right)$ ***(M1)(A1)***

Note: Award ***M1*** for the sum of two products.

$= \frac{31}{63}\ (= 0.4920\ldots)$ ***A1***

[3 marks]

b. Use of $\mathrm{P}(S|F) = \frac{\mathrm{P}(S \cap F)}{\mathrm{P}(F)}$ to obtain $\mathrm{P}(S|F) = \frac{\frac{1}{7} \times \frac{7}{9}}{\frac{31}{63}}$. ***M1***

Note: Award ***M1*** only if the numerator results from the product of two probabilities.

$= \frac{7}{31}\ (= 0.2258\ldots)$ ***A1***

[2 marks]

Examiners report

a. Both parts were very well done. In part (a), most candidates successfully used a tree diagram.

b. Both parts were very well done. In part (b), most candidates correctly used conditional probability considerations.

The marks obtained by a group of students in a class test are shown below.

Marks	Frequency
5	6
6	k
7	3
8	1
9	2
10	1

Given the mean of the marks is 6.5, find the value of k.

Markscheme

$\frac{5 \times 6 + 6k + 7 \times 3 + 8 \times 1 + 9 \times 2 + 10 \times 1}{13 + k} = 6.5$ (or equivalent) ***(M1)(A1)(A1)***

Note: Award ***(M1)(A1)*** for correct numerator, and ***(A1)*** for correct denominator.

$0.5k = 2.5 \Rightarrow k = 5$ ***A1***

[4 marks]

Examiners report

The question was well done generally as one would expect.

A Chocolate Shop advertises free gifts to customers that collect three vouchers. The vouchers are placed at random into 10% of all chocolate bars sold at this shop. Kati buys some of these bars and she opens them one at a time to see if they contain a voucher. Let $\mathrm{P}(X=n)$ be the probability that Kati obtains her third voucher on the $n\mathrm{th}$ bar opened.

(It is assumed that the probability that a chocolate bar contains a voucher stays at 10% throughout the question.)

It is given that $\mathrm{P}(X=n)=\frac{n^2+an+b}{2000}\times 0.9^{n-3}$ for $n\geqslant 3,\ n\in\mathbb{N}$.

Kati's mother goes to the shop and buys x chocolate bars. She takes the bars home for Kati to open.

a. Show that $\mathrm{P}(X=3)=0.001$ and $\mathrm{P}(X=4)=0.0027$. [3]

b. Find the values of the constants a and b. [5]

c. Deduce that $\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}=\frac{0.9(n-1)}{n-3}$ for $n>3$. [4]

d. (i) Hence show that X has two modes m_1 and m_2. [5]

(ii) State the values of m_1 and m_2.

e. Determine the minimum value of x such that the probability Kati receives at least one free gift is greater than 0.5. [3]

Markscheme

a. $\mathrm{P}(X=3)=(0.1)^3$ ***A1***

$=0.001$ ***AG***

$\mathrm{P}(X=4)=\mathrm{P}(VV\bar{V}V)+\mathrm{P}(V\bar{V}VV)+\mathrm{P}(\bar{V}VVV)$ ***(M1)***

$=3\times(0.1)^3\times 0.9$ (or equivalent) ***A1***

$=0.0027$ ***AG***

[3 marks]

b. **METHOD 1**

attempting to form equations in a and b ***M1***

$\frac{9+3a+b}{2000}=\frac{1}{1000}\ (3a+b=-7)$ ***A1***

$\frac{16+4a+b}{2000}\times\frac{9}{10}=\frac{27}{10\,000}\ (4a+b=-10)$ ***A1***

attempting to solve simultaneously ***(M1)***

$a=-3,\ b=2$ ***A1***

METHOD 2

$\mathrm{P}(X=n)=\binom{n-1}{2}\times 0.1^3\times 0.9^{n-3}$ ***M1***

$= \frac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$ ***(M1)A1***

$= \frac{n^2-3n+2}{2000} \times 0.9^{n-3}$ ***A1***

$a = -3, b = 2$ ***A1***

Note: Condone the absence of 0.9^{n-3} in the determination of the values of a and b.

[5 marks]

c. **METHOD 1**

EITHER

$\mathrm{P}(X = n) = \frac{n^2-3n+2}{2000} \times 0.9^{n-3}$ ***(M1)***

OR

$\mathrm{P}(X = n) = \binom{n-1}{2} \times 0.1^3 \times 0.9^{n-3}$ ***(M1)***

THEN

$= \frac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$ ***A1***

$\mathrm{P}(X = n-1) = \frac{(n-2)(n-3)}{2000} \times 0.9^{n-4}$ ***A1***

$\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)} = \frac{(n-1)(n-2)}{(n-2)(n-3)} \times 0.9$ ***A1***

$= \frac{0.9(n-1)}{n-3}$ ***AG***

METHOD 2

$\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)} = \frac{\frac{n^2-3n+2}{2000} \times 0.9^{n-3}}{\frac{(n-1)^2-3(n-1)+2}{2000} \times 0.9^{n-4}}$ ***(M1)***

$= \frac{0.9(n^2-3n+2)}{(n^2-5n+6)}$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$= \frac{0.9(n-1)(n-2)}{(n-2)(n-3)}$ ***A1***

$= \frac{0.9(n-1)}{n-3}$ ***AG***

[4 marks]

d. (i) attempting to solve $\frac{0.9(n-1)}{n-3} = 1$ for n ***M1***

$n = 21$ ***A1***

$\frac{0.9(n-1)}{n-3} < 1 \Rightarrow n > 21$ ***R1***

$\frac{0.9(n-1)}{n-3} > 1 \Rightarrow n < 21$ ***R1***

X has two modes ***AG***

Note: Award ***R1R1*** for a clearly labelled graphical representation of the two inequalities (using $\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}$).

(ii) the modes are 20 and 21 ***A1***

[5 marks]

e. **METHOD 1**

$Y \sim \mathrm{B}(x,\ 0.1)$ ***(A1)***

attempting to solve $\mathrm{P}(Y \geqslant 3) > 0.5$ (or equivalent *eg* $1 - \mathrm{P}(Y \leqslant 2) > 0.5$) for x ***(M1)***

Note: Award ***(M1)*** for attempting to solve an equality (obtaining $x = 26.4$).

$x = 27$ ***A1***

METHOD 2

$\sum\limits_{n=0}^{x} \mathrm{P}(X = n) > 0.5$ ***(A1)***

attempting to solve for x ***(M1)***

$x = 27$ ***A1***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The number of taxis arriving at Cardiff Central railway station can be modelled by a Poisson distribution. During busy periods of the day, taxis arrive at a mean rate of 5.3 taxis every 10 minutes. Let T represent a random 10 minute busy period.

a.i. Find the probability that exactly 4 taxis arrive during T. [2]

a.ii. Find the most likely number of taxis that would arrive during T. [2]

a.iii. Given that more than 5 taxis arrive during T, find the probability that exactly 7 taxis arrive during T. [3]

b. During quiet periods of the day, taxis arrive at a mean rate of 1.3 taxis every 10 minutes. [6]

Find the probability that during a period of 15 minutes, of which the first 10 minutes is busy and the next 5 minutes is quiet, that exactly 2 taxis arrive.

Markscheme

a.i. $X \sim \mathrm{Po}\,(5.3)$

$\mathrm{P}\,(X = 4) = \mathrm{e}^{-5.3}\,\frac{5.3^4}{4!}$ ***(M1)***

= 0.164 ***A1***

[2 marks]

a.ii. **METHOD 1**

listing probabilities (table or graph) ***M1***

mode $X = 5$ (with probability 0.174) ***A1***

Note: Award ***M0A0*** for 5 (taxis) or mode = 5 with no justification.

METHOD 2

mode is the integer part of mean ***R1***

E(X) = 5.3 ⇒ mode = 5 ***A1***

Note: Do not allow ***R0A1***.

[2 marks]

a.iii. attempt at conditional probability ***(M1)***

$\frac{\mathrm{P}(X=7)}{\mathrm{P}(X\geqslant 6)}$ or equivalent $\left(=\frac{0.1163\ldots}{0.4365\ldots}\right)$ ***A1***

= 0.267 ***A1***

[3 marks]

b. ***METHOD 1***

the possible arrivals are (2,0), (1,1), (0,2) ***(A1)***

$Y \sim \mathrm{Po}\,(0.65)$ ***A1***

attempt to compute, using sum and product rule, ***(M1)***

0.070106… × 0.52204… + 0.026455… × 0.33932… + 0.0049916… × 0.11028… ***(A1)(A1)***

Note: Award ***A1*** for one correct product and ***A1*** for two other correct products.

= 0.0461 ***A1***

[6 marks]

METHOD 2

recognising a sum of 2 independent Poisson variables *eg* $Z = X + Y$ ***R1***

$\lambda = 5.3 + \frac{1.3}{2}$

P(Z = 2) = 0.0461 ***(M1)A3***

[6 marks]

Examiners report

a.i. [N/A]

a.ii. [N/A]

a.iii. [N/A]

b. [N/A]

a. (i) Express the sum of the first n positive odd integers using sigma notation. [4]

(ii) Show that the sum stated above is n^2.

(iii) Deduce the value of the difference between the sum of the first 47 positive odd integers and the sum of the first 14 positive odd integers.

b. A number of distinct points are marked on the circumference of a circle, forming a polygon. Diagonals are drawn by joining all pairs of non-adjacent points. [7]

(i) Show on a diagram all diagonals if there are 5 points.

(ii) Show that the number of diagonals is $\frac{n(n-3)}{2}$ if there are n points, where $n > 2$.

(iii) Given that there are more than one million diagonals, determine the least number of points for which this is possible.

c. The random variable $X \sim B(n,\ p)$ has mean 4 and variance 3. [8]

(i) Determine n and p.

(ii) Find the probability that in a single experiment the outcome is 1 or 3.

Markscheme

a. (i) $\sum\limits_{k=1}^{n}(2k-1)$ (or equivalent) ***A1***

Note: Award ***A0*** for $\sum\limits_{n=1}^{n}(2n-1)$ or equivalent.

(ii) **EITHER**

$2 \times \frac{n(n+1)}{2} - n$ ***M1A1***

OR

$\frac{n}{2}(2+(n-1)2)$ (using $S_n = \frac{n}{2}(2u_1+(n-1)d)$) ***M1A1***

OR

$\frac{n}{2}(1+2n-1)$ (using $S_n = \frac{n}{2}(u_1+u_n)$) ***M1A1***

THEN

$= n^2$ ***AG***

(iii) $47^2 - 14^2 = 2013$ ***A1***

[4 marks]

b. (i) **EITHER**

a pentagon and five diagonals ***A1***

OR

five diagonals (circle optional) ***A1***

(ii) Each point joins to $n-3$ other points. ***A1***

a correct argument for $n(n-3)$ ***R1***

a correct argument for $\frac{n(n-3)}{2}$ ***R1***

(iii) attempting to solve $\frac{1}{2}n(n-3) > 1\,000\,000$ for n. ***(M1)***

$n > 1415.7$ ***(A1)***

$n = 1416$ ***A1***

[7 marks]

c. (i) $np = 4$ and $npq = 3$ ***(A1)***

attempting to solve for n and p ***(M1)***

$n = 16$ and $p = \frac{1}{4}$ ***A1***

(ii) $X \sim B(16, 0.25)$ ***(A1)***

$P(X = 1) = 0.0534538...(= \binom{16}{1}(0.25)(0.75)^{15})$ ***(A1)***

$P(X = 3) = 0.207876...(= \binom{16}{3}(0.25)^3(0.75)^{13})$ ***(A1)***

$\mathrm{P}(X = 1) + \mathrm{P}(X = 3)$ ***(M1)***

$= 0.261$ ***A1***

[8 marks]

Examiners report

a. In part (a) (i), a large number of candidates were unable to correctly use sigma notation to express the sum of the first n positive odd integers. Common errors included summing $2n - 1$ from 1 to n and specifying sums with incorrect limits. Parts (a) (ii) and (iii) were generally well done.

b. Parts (b) (i) and (iii) were generally well done. In part (b) (iii), many candidates unnecessarily simplified their quadratic when direct GDC use could have been employed. A few candidates gave $n > 1416$ as their final answer. While some candidates displayed sound reasoning in part (b) (ii), many candidates unfortunately adopted a 'proof by example' approach.

c. Part (c) was generally well done. In part (c) (ii), some candidates multiplied the two probabilities rather than adding the two probabilities.

Tim goes to a popular restaurant that does not take any reservations for tables. It has been determined that the waiting times for a table are normally distributed with a mean of 18 minutes and standard deviation of 4 minutes.

(a) Tim says he will leave if he is not seated at a table within 25 minutes of arriving at the restaurant. Find the probability that Tim will leave without being seated.

(b) Tim has been waiting for 15 minutes. Find the probability that he will be seated within the next five minutes.

Markscheme

the waiting time, $X \sim N(18, 4^2)$

(a) $\mathrm{P}(X > 25) = 0.0401$ ***(M1)A1***

(b) $\mathrm{P}(X < 20 | X > 15)$ ***(A1)***

$= \frac{\mathrm{P}(15 < X < 20)}{\mathrm{P}(X > 15)}$ ***(A1)***

Note: Only one of the above ***A1*** marks can be implied.

$= \frac{0.4648...}{0.7733...} = 0.601$ ***(M1)A1***

[6 marks]

Examiners report

Part (a) was well answered, whilst few candidates managed to correctly use conditional probability for part (b).

The random variable X has a Poisson distribution with mean μ.

Given that $\mathrm{P}(X = 2) + \mathrm{P}(X = 3) = \mathrm{P}(X = 5)$,

(a) find the value of μ;

(b) find the probability that X lies within one standard deviation of the mean.

Markscheme

(a) $\frac{\mu^2 e^{-\mu}}{2!} + \frac{\mu^3 e^{-\mu}}{3!} = \frac{\mu^5 e^{-\mu}}{5!}$ ***(M1)***

$\frac{\mu^2}{2} + \frac{\mu^3}{6} - \frac{\mu^5}{120} = 0$

$\mu = 5.55$ ***A1***

[2 marks]

(b) $\sigma = \sqrt{5.55\ldots} = 2.35598\ldots$ ***(M1)***

$\mathrm{P}(3.19 \leqslant X \leqslant 7.9)$

$\mathrm{P}(4 \leqslant X \leqslant 7)$

$= 0.607$ ***A1***

[2 marks]

Total [4 marks]

Examiners report

[N/A]

Casualties arrive at an accident unit with a mean rate of one every 10 minutes. Assume that the number of arrivals can be modelled by a Poisson distribution.

(a) Find the probability that there are no arrivals in a given half hour period.

(b) A nurse works for a two hour period. Find the probability that there are fewer than ten casualties during this period.

(c) Six nurses work consecutive two hour periods between 8am and 8pm. Find the probability that no more than three nurses have to attend to less than ten casualties during their working period.

(d) Calculate the time interval during which there is a 95 % chance of there being at least two casualties.

Markscheme

Note: Accept exact answers in parts (a) to (c).

(a) number of patients in 30 minute period = X ***(A1)***

$X \sim \text{Po}(3)$ ***(M1)A1***

[3 marks]

(b) number of patients in working period = $\boldsymbol{Y}$ ***(A1)***

$Y \sim \text{Po}(12)$ ***(M1)A1***

[3 marks]

(c) number of working period with less than 10 patients = W ***(M1)(A1)***

$W \sim \text{B}(6,\ 0.2424\ldots)$ ***(M1)A1***

[4 marks]

(d) number of patients in t minute interval = X

$X \sim \text{Po}(T)$

$\text{P}(X \geqslant 2) = 0.95$

$\text{P}(X = 0) + \text{P}(X = 1) = 0.05$ ***(M1)(A1)***

$\text{e}^{-T}(1 + T) = 0.05$ ***(M1)***

$T = 4.74$ ***(A1)***

t = 47.4 minutes ***A1***

[5 marks]

Total [15 marks]

Examiners report

Parts (a) and (b) were well answered, but many students were unable to recognise the Binomial distribution in part (c) and were unable to form the correct equation in part (d). There were many accuracy errors in this question.

A small car hire company has two cars. Each car can be hired for one whole day at a time. The rental charge is US$60 per car per day. The number of requests to hire a car for one whole day may be modelled by a Poisson distribution with mean 1.2.

a. Find the probability that on a particular weekend, three requests are received on Saturday and none are received on Sunday. [2]

b. Over a weekend of two days, it is given that a total of three requests are received. [5]

Find the expected total rental income for the weekend.

Markscheme

a. $X \sim \mathrm{Po}(1.2)$

$\mathrm{P}(X=3) \times \mathrm{P}(X=0)$ ***(M1)***

$= 0.0867\ldots \times 0.3011\ldots$

$= 0.0261$ ***A1***

[2 marks]

b. Three requests over two days can occur as (3, 0), (0, 3), (2, 1) or (1, 2). ***R1***

using conditional probability, for example

$\frac{\mathrm{P}(3,\,0)}{\mathrm{P}(3\text{ requests, }m{=}2.4)} = 0.125$ or $\frac{\mathrm{P}(2,\,1)}{\mathrm{P}(3\text{ requests, }m{=}2.4)} = 0.375$ ***M1A1***

expected income is

$2 \times 0.125 \times \text{US\$}120 + 2 \times 0.375 \times \text{US\$}180$ ***M1***

Note: Award ***M1*** for attempting to find the expected income including both (3, 0) and (2, 1) cases.

$= \text{US\$}30 + \text{US\$}135$

$= \text{US\$}165$ ***A1***

[5 marks]

Examiners report

a. Part (a) was generally well done although a number of candidates added the two probabilities rather than multiplying the two probabilities. A number of candidates specified the required probability correct to two significant figures only.

b. Part (b) challenged most candidates with only a few candidates able to correctly employ a conditional probability argument.

Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of 998 ml and a standard deviation of 2.5 ml.

(a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212.

(b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water.

(c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water, is greater than 0.99.

(d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B.

(e) The company that makes the mineral water receives, on average, m phone calls every 10 minutes. The number of phone calls, X , follows a Poisson distribution such that $\mathrm{P}(X=2)=\mathrm{P}(X=3)+\mathrm{P}(X=4)$.

(i) Find the value of m .

(ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period.

Markscheme

(a) $X \sim \mathrm{N}(998, 2.5^2)$ ***M1***

$\mathrm{P}(X>1000)=0.212$ ***AG***

[1 mark]

(b) $X \sim \mathrm{B}(5, 0.2119...)$

evidence of binomial ***(M1)***

$\mathrm{P}(X=3)=\binom{5}{3}(0.2119...)^3(0.7881...)^2=0.0591$ (accept 0.0592) ***(M1)A1***

[3 marks]

(c) $\mathrm{P}(X \geqslant 1)=1-\mathrm{P}(X=0)$ ***(M1)***

$1-(0.7881...)^n>0.99$

$(0.7881...)^n<0.01$ ***A1***

Note: Award ***A1*** for line 2 or line 3 or equivalent.

$n>19.3$ ***(A1)***
minimum number of bottles required is 20 ***A1N2***

[4 marks]

(d) $\frac{996-\mu}{\sigma}=-1.1998$ (accept 1.2) ***M1A1***

$\frac{1000-\mu}{\sigma}=0.3999$ (accept 0.4) ***M1A1***

$\mu=999(ml)$, $\sigma=2.50(ml)$ ***A1A1***

[6 marks]

(e) (i) $\frac{\mathrm{e}^{-m}m^2}{2!}=\frac{\mathrm{e}^{-m}m^3}{3!}+\frac{\mathrm{e}^{-m}m^4}{4!}$ ***M1A1***

$\frac{m^2}{2}=\frac{m^3}{6}+\frac{m^4}{24}$

$12m^2-4m^3-m^4=0$ ***(A1)***

$m=-6, 0, 2$

$\Rightarrow m=2$ ***A1N2***

(ii) $\mathrm{P}(X>2)=1-\mathrm{P}(X \leqslant 2)$ ***(M1)***

$=1-\mathrm{P}(X=0)-\mathrm{P}(X=1)-\mathrm{P}(X=2)$

$= 1 - e^{-2} - 2e^{-2} - \frac{2^2 e^{-2}}{2!}$

$= 0.323$ ***A1***

[6 marks]

Total [20 marks]

Examiners report

This was the best done of the section B questions, with the majority of candidates making the correct choice of probability distribution for each part. The main sources of errors: (b) missing out the binomial coefficient in the calculation; (c) failure to rearrange 'at least one bottle' in terms of the probability of obtaining no bottles; (d) using 1.2 rather than -1.2 in the inverse Normal or not performing an inverse Normal at all; (e)(ii) misinterpreting 'more than two'.

The lifts in the office buildings of a small city have occasional breakdowns. The breakdowns at any given time are independent of one another and can be modelled using a Poisson Distribution with mean 0.2 per day.

(a) Determine the probability that there will be exactly four breakdowns during the month of June (June has 30 days).

(b) Determine the probability that there are more than 3 breakdowns during the month of June.

(c) Determine the probability that there are no breakdowns during the first five days of June.

(d) Find the probability that the first breakdown in June occurs on June 3^{rd}.

(e) It costs 1850 Euros to service the lifts when they have breakdowns. Find the expected cost of servicing lifts for the month of June.

(f) Determine the probability that there will be no breakdowns in exactly 4 out of the first 5 days in June.

Markscheme

(a) mean for 30 days: $30 \times 0.2 = 6$. ***(A1)***

$\mathrm{P}(X = 4) = \frac{6^4}{4!}e^{-6} = 0.134$ ***(M1)A1 N3***

[3 marks]

(b) $\mathrm{P}(X > 3) = 1 - \mathrm{P}(X \leqslant 3) = 1 - e^{-6}(1 + 6 + 18 + 36) = 0.849$ ***(M1)A1 N2***

[2 marks]

(c) **EITHER**

mean for five days: $5 \times 0.2 = 1$ ***(A1)***

$\mathrm{P}(X = 0) = e^{-1} \quad (= 0.368)$ ***A1 N2***

OR

mean for one day: 0.2 ***(A1)***

$\mathrm{P}(X = 0) = (e^{-0.2})^5 = e^{-1} \quad (= 0.368)$ ***A1 N2***

[2 marks]

(d) Required probability $= \mathrm{e}^{-0.2} \times \mathrm{e}^{-0.2} \times (1 - \mathrm{e}^{-0.2})$ ***M1A1***

$= 0.122$ ***A1*** ***N3***

[3 marks]

(e) Expected cost is $1850 \times 6 = 11\,100$ Euros ***A1***

[1 mark]

(f) On any one day $\mathrm{P}(X = 0) = \mathrm{e}^{-0.2}$

Therefore, $\binom{5}{1}(\mathrm{e}^{-0.2})^4(1 - \mathrm{e}^{-0.2}) = 0.407$ ***M1A1*** ***N2***

[2 marks]

Total [13 marks]

Examiners report

Many candidates showed familiarity with the Poisson Distribution. Parts (a), (b), and (c) were straightforward, as long as candidates multiplied 0.2 by 30 to get the mean. Part (e) was answered successfully by most candidates. Parts (d) and (f) were done very poorly. In part (d), most candidates calculated $\mathrm{P}(X = 1)$ rather than $\mathrm{P}(X \leqslant 1)$. Although some candidates realized the need for the Binomial in part (e), some incorrectly used 0.8 and 0.2.

The wingspans of a certain species of bird can be modelled by a normal distribution with mean 60.2 cm and standard deviation 2.4 cm.

According to this model, 99% of wingspans are greater than x cm.

a. Find the value of x. [2]

b. In a field experiment, a research team studies a large sample of these birds. The wingspans of each bird are measured correct to the nearest 0.1 cm. [3]

Find the probability that a randomly selected bird has a wingspan measured as 60.2 cm.

Markscheme

a. $\mathrm{P}(X > x) = 0.99 \quad (= \mathrm{P}(X < x) = 0.01)$ ***(M1)***

$\Rightarrow x = 54.6 \text{ (cm)}$ ***A1***

[2 marks]

b. $\mathrm{P}(60.15 \leq X \leq 60.25)$ ***(M1)(A1)***

$= 0.0166$ ***A1***

[3 marks]

Total [5 marks]

Examiners report

a. Many candidates did not use the symmetry of the normal curve correctly. Many, for example, calculated the value of x for which $\mathrm{P}(X < x) = 0.99$ rather than $\mathrm{P}(X < x) = 0.01$.

b. Most candidates did not recognize that the required probability interval was $\mathrm{P}(60.15 \le X \le 60.25)$. A large number of candidates simply stated that $\mathrm{P}(X = 60.2) = 0.166$. Some candidates used $\mathrm{P}(60.1 \le X \le 60.3)$ while a number of candidates bizarrely used probability intervals not centred on 60.2, for example, $\mathrm{P}(60.15 \le X \le 60.24)$.

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a. Find the term in x^5 in the expansion of $(3x + A)(2x + B)^6$. [4]

b. Mina and Norbert each have a fair cubical die with faces labelled 1, 2, 3, 4, 5 and 6; they throw it to decide if they are going to eat a cookie. [4]

Mina throws her die just once and she eats a cookie if she throws a four, a five or a six.

Norbert throws his die six times and each time eats a cookie if he throws a five or a six.

Calculate the probability that five cookies are eaten.

Markscheme

a. $\left(A\binom{6}{5}2^5B + 3\binom{6}{4}2^4B^2\right)x^5$ ***M1A1A1***

$= \left(192AB + 720B^2\right)x^5$ ***A1***

[4 marks]

b. **METHOD 1**

$x = \frac{1}{6},\ A = \frac{3}{6}\left(= \frac{1}{2}\right),\ B = \frac{4}{6}\left(= \frac{2}{3}\right)$ ***A1A1A1***

probability is $\frac{4}{81}$ $(= 0.0494)$ ***A1***

METHOD 2

P (5 eaten) =P (M eats 1) P (N eats 4) + P (M eats 0) P (N eats 5) ***(M1)***

$= \frac{1}{2}\binom{6}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + \frac{1}{2}\binom{6}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)$ ***(A1)(A1)***

$= \frac{4}{81}$ $(= 0.0494)$ ***A1***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

Consider the data set $\{2,\ x,\ y,\ 10,\ 17\}$, $x,\ y \in \mathbb{Z}^+$ and $x < y$.

The mean of the data set is 8 and its variance is 27.6.

Find the value of x and the value of y.

Markscheme

use of $\mu = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ to obtain $\frac{2+x+y+10+17}{5} = 8$ ***(M1)***

$x + y = 11$ ***A1***

EITHER

use of $\sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$ to obtain $\frac{(-6)^2+(x-8)^2+(y-8)^2+2^2+9^2}{5} = 27.6$ ***(M1)***

$(x-8)^2 + (y-8)^2 = 17$ ***A1***

OR

use of $\sigma^2 = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$ to obtain $\frac{2^2+x^2+y^2+10^2+17^2}{5} - 8^2 = 27.6$ ***(M1)***

$x^2 + y^2 = 65$ ***A1***

THEN

attempting to solve the two equations ***(M1)***

$x = 4$ and $y = 7$ $(\text{only as } x < y)$***A1*** ***N4***

Note: Award ***A0*** for $x = 7$ and $y = 4$.

Note: Award ***(M1)A1(M0)A0(M1)A1*** for $x + y = 11 \Rightarrow x = 4$ and $y = 7$.

[6 marks]

Examiners report

Reasonably well done. Most candidates were able to obtain $x + y = 11$. Most manipulation errors occurred when candidates attempted to form the variance equation in terms of x and y. Some candidates did not apply the condition $x < y$ when determining their final answer.

A discrete random variable X has a probability distribution given in the following table.

x	0.5	1.5	2.5	3.5	4.5	5.5
$P(X = x)$	0.15	0.21	p	q	0.13	0.07

(a) $E(X) = 2.61$, determine the value of p and of q.

(b) Calculate $\text{Var}(X)$ to three significant figures.

Markscheme

(a) $p+q=0.44$ ***A1***

$2.5p+3.5q=1.25$ ***(M1)A1***

$p=0.29,\ q=0.15$ ***A1***

(b) use of $\mathrm{Var}(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2$ ***(M1)***

$\mathrm{Var}(X)=2.10$ ***A1***

[6 marks]

Examiners report

An easy question, well answered by most candidates. For the others it was disappointing that many did not use the fact that the probabilities add to unity.

The number of bananas that Lucca eats during any particular day follows a Poisson distribution with mean 0.2.

a. Find the probability that Lucca eats at least one banana in a particular day. [2]

b. Find the expected number of weeks in the year in which Lucca eats no bananas. [4]

Markscheme

a. let X be the number of bananas eaten in one day

$X \sim \mathrm{Po}(0.2)$

$\mathrm{P}(X \geqslant 1)=1-\mathrm{P}(X=0)$ ***(M1)***

$=0.181\ (=1-\mathrm{e}^{-0.2})$ ***A1***

[2 marks]

b. **EITHER**

let Y be the number of bananas eaten in one week

$\mathrm{Y} \sim \mathrm{Po}(1.4)$ ***(A1)***

$\mathrm{P}(Y=0)=0.246596\ldots\ (=\mathrm{e}^{-1.4})$ ***(A1)***

OR

let Z be the number of days in one week at least one banana is eaten

$Z \sim \mathrm{B}(7,\ 0.181\ldots)$ ***(A1)***

$\mathrm{P}(Z=0)=0.246596\ldots$ ***(A1)***

THEN

$52 \times 0.246596\ldots$ ***(M1)***

$=12.8\ (=52\mathrm{e}^{-1.4})$ ***A1***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

After being sprayed with a weedkiller, the survival time of weeds in a field is normally distributed with a mean of 15 days.

(a) If the probability of survival after 21 days is 0.2 , find the standard deviation of the survival time.

When another field is sprayed, the survival time of weeds is normally distributed with a mean of 18 days.

(b) If the standard deviation of the survival time is unchanged, find the probability of survival after 21 days.

Markscheme

(a) required to solve P $\left(Z < \frac{21-15}{\sigma}\right) = 0.8$ ***(M1)***

$\frac{6}{\sigma} = 0.842\ldots$ (or equivalent) ***(M1)***

$\Rightarrow \sigma = 7.13$ (days) ***A1 N1***

(b) P(survival after 21 days) = 0.337 ***(M1)A1***

[5 marks]

Examiners report

A straightforward Normal distribution problem, but many candidates confused the *z* value with the probability.

The random variable *X* has a normal distribution with mean $\mu = 50$ and variance $\sigma^2 = 16$.

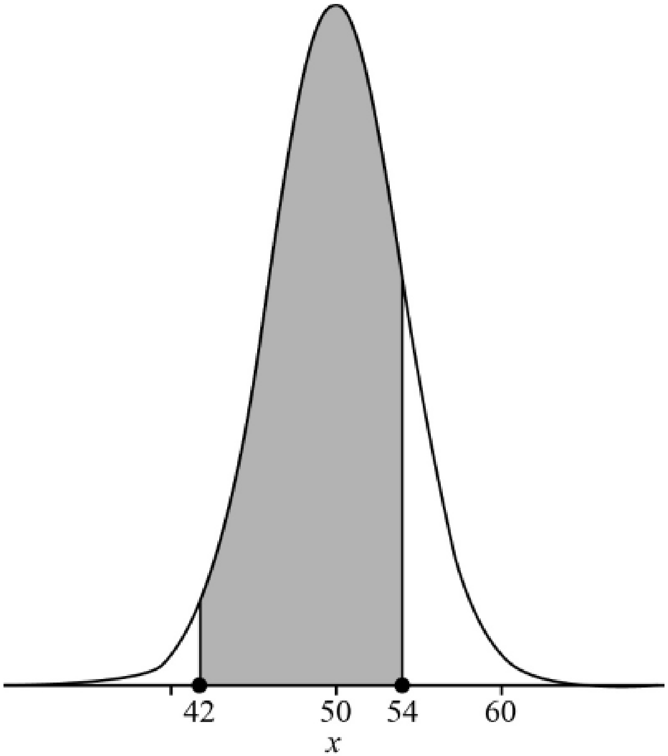
a. Sketch the probability density function for *X*, and shade the region representing P($\mu - 2\sigma < X < \mu + \sigma$). [2]

b. Find the value of P($\mu - 2\sigma < X < \mu + \sigma$). [2]

c. Find the value of *k* for which P($\mu - k\sigma < X < \mu + k\sigma$) = 0.5. [2]

Markscheme

a.



normal curve centred on 50 ***A1***

vertical lines at $x = 42$ and $x = 54$, with shading in between ***A1***

[2 marks]

b. P(42 < *X* < 54) (= P(– 2 < *Z* < 1)) ***(M1)***

= 0.819 ***A1***

[2 marks]

c. $P(\mu - k\sigma < X < \mu + k\sigma) = 0.5 \Rightarrow P(X < \mu + k\sigma) = 0.75$ ***(M1)***

$k = 0.674$ ***A1***

Note: Award ***M1A0*** for $k = -0.674$.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Tim throws two identical fair dice simultaneously. Each die has six faces: two faces numbered 1, two faces numbered 2 and two faces numbered 3.

His score is the sum of the two numbers shown on the dice.

(a) (i) Calculate the probability that Tim obtains a score of 6.

(ii) Calculate the probability that Tim obtains a score of at least 3.

Tim plays a game with his friend Bill, who also has two dice numbered in the same way. Bill's score is the sum of the two numbers shown on his dice.

(b) (i) Calculate the probability that Tim and Bill **both** obtain a score of 6.

(ii) Calculate the probability that Tim and Bill obtain the same score.

(c) Let X denote the largest number shown on the four dice.

(i) Show that $\mathrm{P}(X \leqslant 2) = \frac{16}{81}$.

(ii) Copy and complete the following probability distribution table.

x	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{81}$		

(iii) Calculate $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$ and hence find $\mathrm{Var}(X)$.

(d) Given that $X = 3$, find the probability that the sum of the numbers shown on the four dice is 8.

Markscheme

+	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

(a) let T be Tim's score

(i) $\mathrm{P}(T = 6) = \frac{11}{9}$ $(= 0.111 \text{ 3 sf})$ ***A1***

(ii) $\mathrm{P}(T \geqslant 3) = 1 - \mathrm{P}(T \leqslant 2) = 1 - \frac{1}{9} = \frac{8}{9}$ $(= 0.889 \text{ 3 sf})$ ***(M1)A1***

[3 marks]

(b) let B be Bill's score

(i) $\mathrm{P}(T = 6 \text{ and } B = 6) = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$ $(= 0.012 \text{ 3 sf})$ ***(M1)A1***

(ii) $\mathrm{P}(B = T) = \mathrm{P}(2)\mathrm{P}(2) + \mathrm{P}(3)\mathrm{P}(3) + \ldots + \mathrm{P}(6)\mathrm{P}(6)$

$= \frac{1}{9} \times \frac{1}{9} + \frac{2}{9} \times \frac{2}{9} + \frac{3}{9} \times \frac{3}{9} + \frac{2}{9} \times \frac{2}{9} + \frac{1}{9} \times \frac{1}{9}$ ***M1***

$= \frac{19}{81}$ $(= 0.235 \text{ 3 sf})$ ***A1***

[4 marks]

(c) (i) **EITHER**

$\mathrm{P}(X \leqslant 2) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ ***M1A1***

because $\mathrm{P}(X \leqslant 2) = \mathrm{P}\left((a,\ b,\ c,\ d) | a,\ b,\ c,\ d = 1,\ 2\right)$ ***R1***

or equivalent

$\mathrm{P}(X \leqslant 2) = \frac{16}{81}$ ***AG***

OR

there are sixteen possible permutations which are

Combinations	Number
1111	1
1112	4
1122	6
1222	4
2222	1

M1A1

Note: This information may be presented in a variety of forms.

$\mathrm{P}(X \leqslant 2) = \frac{1+4+6+4+1}{81}$ ***A1***

$= \frac{16}{81}$ ***AG***

(ii)

x	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{81}$	$\frac{15}{81}$	$\frac{65}{81}$

A1A1

(iii) $\mathrm{E}(X) = \sum\limits_{x=1}^{3} x\mathrm{P}(X = x)$ ***(M1)***

$= \frac{1}{81} + \frac{30}{81} + \frac{195}{81}$

$= \frac{226}{81}$ (2.79 to 3 sf) ***A1***

$\mathrm{E}(X^2) = \sum\limits_{x=1}^{3} x^2\mathrm{P}(X = x)$

$= \frac{1}{81} + \frac{60}{81} + \frac{585}{81}$

$= \frac{646}{81}$ (7.98 to 3 sf) ***A1***

$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$ ***(M1)***

$= 0.191$ (3 sf) ***A1***

Note: Award ***M1A0*** for answers obtained using rounded values ($e.g.$ $\mathrm{Var}(X) = 0.196$).

[10 marks]

(d)

Combinations	Number
3311	6
3221	12

$\mathrm{P}\left(\text{total is } 8 \cap (X = 3)\right) = \frac{18}{81}$ ***M1A1***

since $\mathrm{P}(X = 3) = \frac{65}{81}$

$\mathrm{P}\left(\text{total is } 8 | (X = 3)\right) = \frac{\mathrm{P}((\text{total is } 8) \cap (X=3))}{\mathrm{P}(X=3)}$ ***M1***

$= \frac{18}{65}$ $(= 0.277)$ ***A1***

[4 marks]

Total [21 marks]

Examiners report

Most candidates with a reasonable understanding of probability managed to answer well parts (a), (b) and some of part (c). However some candidates did not realize that different scores were not equally likely which lead to incorrect answers in several parts. Surprisingly, many candidates completed the table in part c) ii) with values that did not add up to 1. Very few candidates answered part (d) well. The enumeration of possible cases was sometimes attempted but with little success.

In each round of two different games Ying tosses three fair coins and Mario tosses two fair coins.

(a) The first game consists of one round. If Ying obtains more heads than Mario, she receives \$5 from Mario. If Mario obtains more heads than Ying, he receives \$10 from Ying. If they obtain the same number of heads, then Mario receives \$2 from Ying. Determine Ying's expected winnings.

(b) They now play the second game, where the winner will be the player who obtains the larger number of heads in a round. If they obtain the same number of heads, they play another round until there is a winner. Calculate the probability that Ying wins the game.

Markscheme

(a) Ying:

Number of heads	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(M1)A1

Mario:

Number of heads	0	1	2
P	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

(M1)A1

$\mathrm{P(Ying\ wins)} = \frac{1}{8} + \frac{3}{8}\left(\frac{2}{4} + \frac{1}{4}\right) + \frac{3}{8} \times \frac{1}{4}$

$= \frac{16}{32}$ ***(M1)A1***

$\mathrm{P(Mario\ wins)} = \frac{1}{4}\left(\frac{3}{8} + \frac{1}{8}\right) + \frac{2}{4} \times \frac{1}{8}$

$= \frac{6}{32}$ ***(M1)A1***

$\mathrm{P(draw)} = 1 - \frac{16}{32} - \frac{6}{32}$

$= \frac{10}{32}$ ***A1***

Ying's winnings:

X	5	−10	−2
P	$\frac{16}{32}$	$\frac{6}{32}$	$\frac{10}{32}$

expected winnings $= 5\left(\frac{16}{32}\right) - 105\left(\frac{6}{32}\right) - 25\left(\frac{10}{32}\right)$ ***M1A1***

$= 0$ ***A1***

[12 marks]

(b) $\mathrm{P(Ying\ wins\ on\ 1st\ round)} = \frac{1}{2}$ ***(A1)***

$\mathrm{P(Ying\ wins\ on\ 2st\ round)} = \frac{5}{16} \times \frac{1}{2}$ ***(M1)(A1)***

$\mathrm{P(Ying\ wins\ on\ 3rd\ round)} = \left(\frac{5}{16}\right)^2 \times \frac{1}{2}$ *etc.* ***(A1)***

$\mathrm{P(Ying\ wins)} = \frac{1}{2} + \frac{5}{16} \times \frac{1}{2} + \left(\frac{5}{16}\right)^2 \times \frac{1}{2} + \ldots$ ***(M1)***

$= \frac{\frac{1}{2}}{1-\frac{5}{16}}$ ***M1A1***

$= \frac{8}{11}$ $(= 0.727)$ ***A1***

[8 marks]

Total [20 marks]

Examiners report

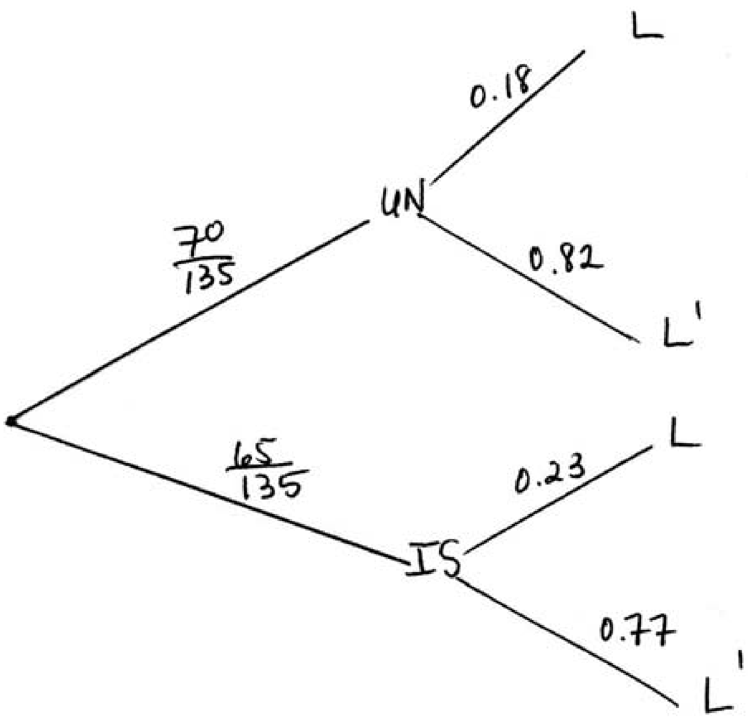
There were some good attempts at this question, but there were also many candidates that were unable to maintain a clearly presented solution and consequently were unable to obtain marks that they should have been able to secure. Those that attempted part (b) usually made a good attempt.

Only two international airlines fly daily into an airport. UN Air has 70 flights a day and IS Air has 65 flights a day. Passengers flying with UN Air have an 18 % probability of losing their luggage and passengers flying with IS Air have a 23 % probability of losing their luggage. You overhear someone in the airport complain about her luggage being lost.

Find the probability that she travelled with IS Air.

Markscheme

METHOD 1



(M1)

Let P(I) be the probability of flying IS Air, P(U) be the probability flying UN Air and P(L) be the probability of luggage lost.

$\mathrm{P}(I|L) = \frac{\mathrm{P}(I \cap L)}{\mathrm{P}(L)}$ $\left(\text{or Bayes' formula , } \mathrm{P}(I|L) = \frac{\mathrm{P}(L|I)\mathrm{P}(I)}{\mathrm{P}(L|I)\mathrm{P}(I)+\mathrm{P}(L|U)\mathrm{P}(U)}\right)$ ***(M1)***

$= \frac{0.23 \times \frac{65}{135}}{0.18 \times \frac{70}{135} + 0.23 \times \frac{65}{135}}$ ***A1A1A1***

$= \frac{299}{551}$ $(= 0.543, \text{ accept } 0.542)$ ***A1***

[6 marks]

METHOD 2

Expected number of suitcases lost by UN Air is $0.18 \times 70 = 12.6$ ***M1A1***

Expected number of suitcases lost by IS Air is $0.23 \times 65 = 14.95$ ***A1***

$\mathrm{P}(I|L) = \frac{14.95}{12.6+14.95}$ ***M1A1***

$= 0.543$ ***A1***

[6 marks]

Examiners report

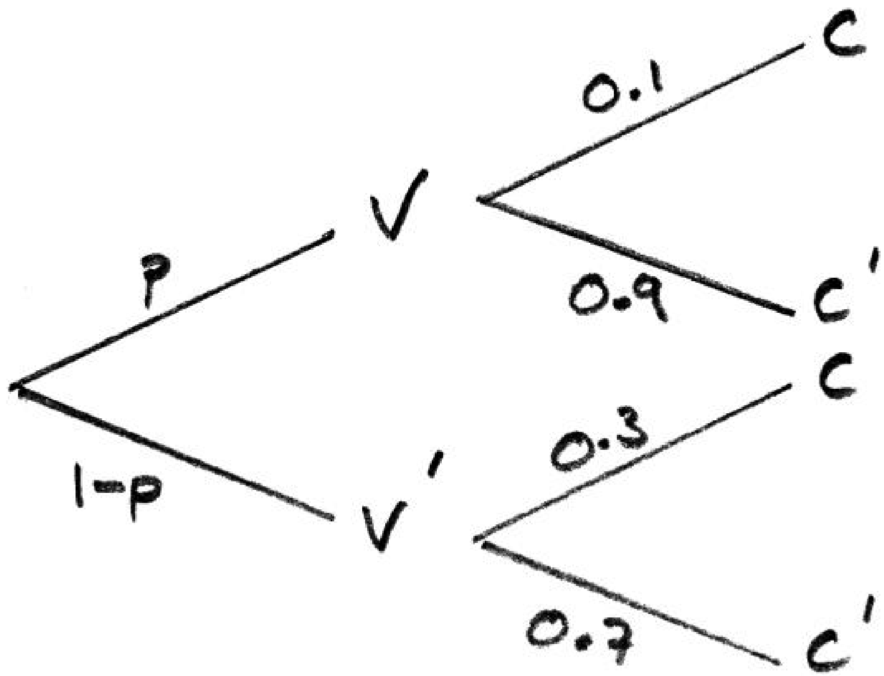
This question was well answered by the majority of candidates. Most candidates used either tree diagrams or expected value methods.

(a) Find the percentage of the population that has been vaccinated.

(b) A randomly chosen person catches the virus. Find the probability that this person has been vaccinated.

Markscheme

(a)



using the law of total probabilities: ***(M1)***

$0.1p + 0.3\,(1 - p) = 0.22$ ***A1***

$0.1p + 0.3 - 0.3p = 0.22$

$0.2p = 0.88$

$p = \frac{0.88}{0.2} = 0.4$

$p = 40\%$ (accept 0.4) ***A1***

(b) required probability $= \frac{0.4 \times 0.1}{0.22}$ ***M1***

$= \frac{2}{11}$ (0.182) ***A1***

[5 marks]

Examiners report

Most candidates who successfully answered this question had first drawn a tree diagram, using a symbol to denote the probability that a randomly chosen person had received the influenza virus. For those who did not draw a tree diagram, there was poor understanding of how to apply the conditional probability formula.

Kathy plays a computer game in which she has to find the path through a maze within a certain time. The first time she attempts the game, the probability of success is known to be 0.75. In subsequent attempts, if Kathy is successful, the difficulty increases and the probability of success is half the probability of success on the previous attempt. However, if she is unsuccessful, the probability of success remains the same. Kathy plays the game three times consecutively.

a. Find the probability that she is successful in all three games. [2]

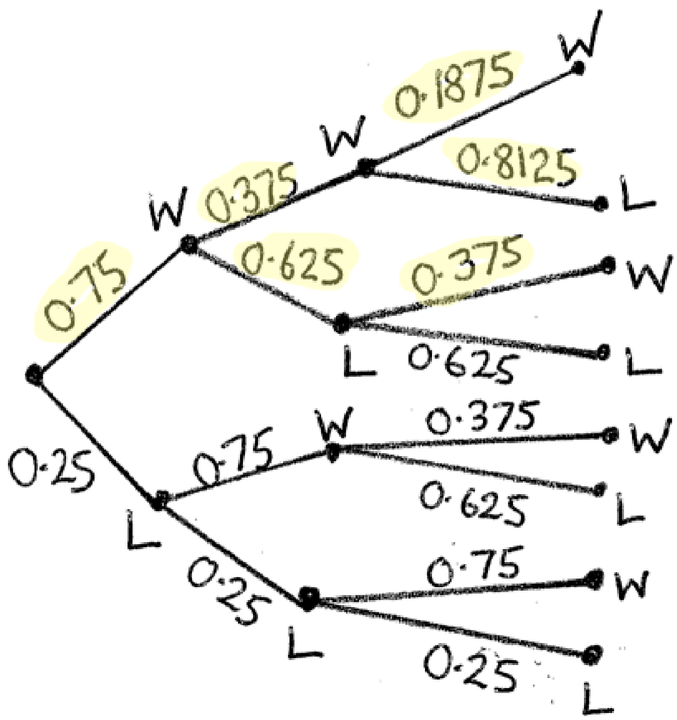
b. Assuming that she is successful in the first game, find the probability that she is successful in exactly two games. [6]

Markscheme

a. $\mathrm{P(WWW)} = 0.75 \times 0.375 \times 0.1875 = 0.0527 \text{ (3sf)} \left(\frac{3}{4} \times \frac{3}{8} \times \frac{3}{16} = \frac{27}{512}\right)$ ***(M1)A1***

[2 marks]

b.



(M1)(A1)

Note: Award ***M1*** for any reasonable attempt to use a tree diagram showing that three games were played (do not award ***M1*** for tree diagrams that only show the first two games) and ***A1*** for the highlighted probabilities.

$\mathrm{P}(\text{wins 2 games} \mid \text{wins first game}) = \frac{\mathrm{P(WWL, WLW)}}{\mathrm{P(wins\ first\ game)}}$ ***(M1)***

$= \frac{0.75 \times 0.375 \times 0.8125 + 0.75 \times 0.625 \times 0.375}{0.75}$ ***(A1)(A1)***

$= 0.539 \text{ (3sf)} \left(\text{or } \frac{69}{128}\right)$ ***A1***

Note: Candidates may use the tree diagram to obtain the answer without using the conditional probability formula, *ie*, $\mathrm{P}(\text{wins 2 games} \mid \text{wins first game}) = 0.375 \times 0.8125 + 0.625 \times 0.375 = 0.539.$

[6 marks]

Examiners report

a. Part (a) was generally successful to most candidates; however the conditional probability was proved difficult to many candidates either because the unconditional probability of two correct games was found or the success in the second and third game was included. Many candidates used a clear tree diagram to calculate the corresponding probabilities. However other candidates frequently tried to do the problem without drawing a tree diagram and often had incorrect probabilities. It was sad to read many answers with probabilities greater than 1.

b. Part (a) was generally successful to most candidates; however the conditional probability was proved difficult to many candidates either because the unconditional probability of two correct games was found or the success in the second and third game was included. Many candidates used a clear tree diagram to calculate the corresponding probabilities. However other candidates frequently tried to do the problem without drawing a tree diagram and often had incorrect probabilities. It was sad to read many answers with probabilities greater than 1.

In a class of 20 students, 12 study Biology, 15 study History and 2 students study neither Biology nor History.

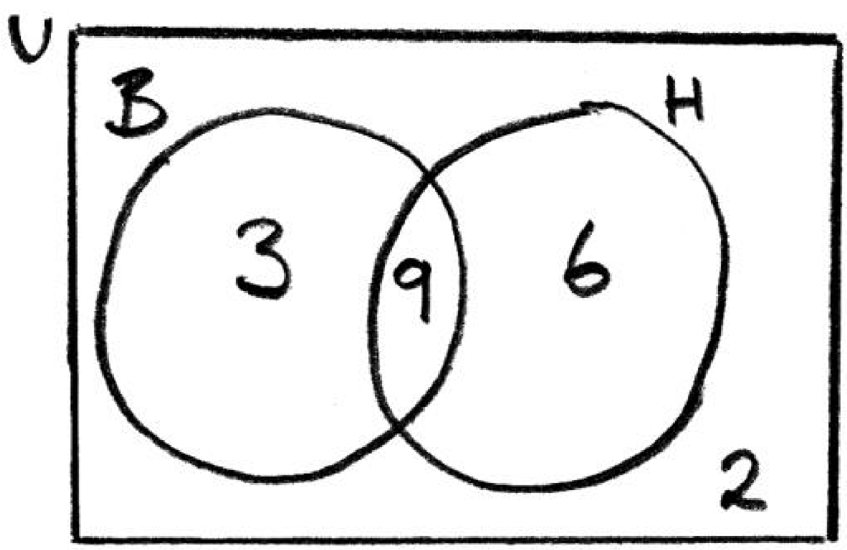
(a) Illustrate this information on a Venn diagram.

(b) Find the probability that a randomly selected student from this class is studying both Biology and History.

(c) Given that a randomly selected student studies Biology, find the probability that this student also studies History.

Markscheme

(a)



A1A1

Note: Award ***A1*** for a diagram with two intersecting regions and at least the value of the intersection.

(b) $\frac{9}{20}$ ***A1***

(c) $\frac{9}{12}\left(=\frac{3}{4}\right)$ ***A1***

[4 marks]

Examiners report

Although this was the best done question on the paper, it was disappointing that a significant number of candidates produced Venn diagrams with key information missing.